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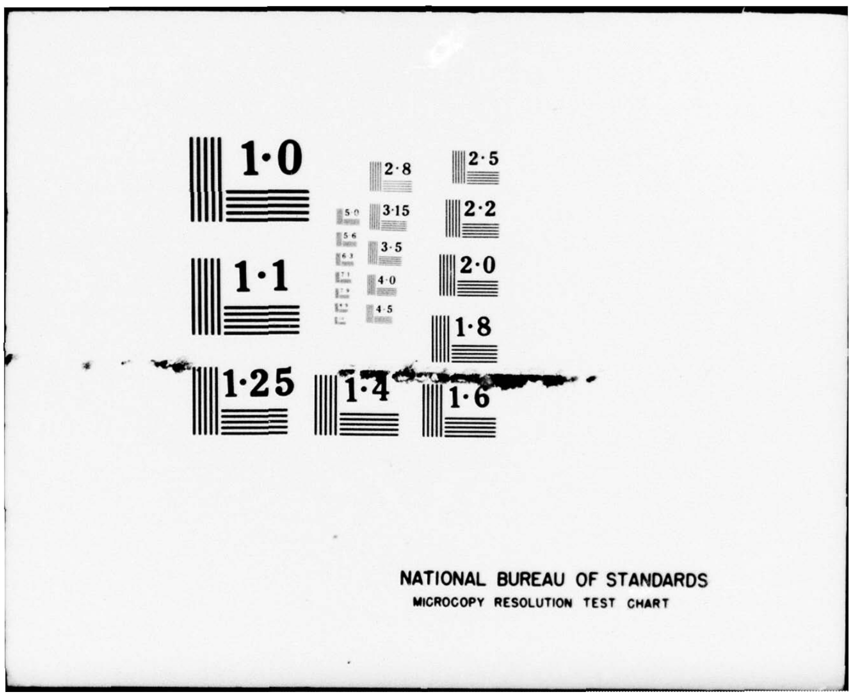
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# A MATHEMATICAL THEORY OF ORGANIZATION (U)

## PART 2: DECENTRALIZED ORGANIZATION

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Results are reported of a second phase in the development of a mathematical theory of organization. Starting from certain assumptions regarding the characteristics of a typical organization member, the theory seeks to derive those of a well-functioning organization. This report deals more particularly with the interplay of the factors which favor, with the ones that militate against, decentralization. The subject is a generalization in several


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respects over one treated in a 1974 ~~an earlier~~ report which dealt with centralized organizations. Theory and practice apparently continue to be in good agreement, a fact which encourages the hope that better theoretical understanding of the functioning of organizations will lead to ways of improving their performance in practice.

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# EXECUTIVE SUMMARY

The purpose of

This report summarizes recent work in the development of a mathematical theory of organization. The purpose of such a theory is to explain in quantitative terms why the structures and operating procedures of organizations in practice are drawn up in the way they now generally are; and It is also to indicate the rationale by which existing organizations can be reviewed and, when possible, revised for better performance. The report treats more specifically the subjects of decentralization and control in organizations which had been omitted in earlier work. It also makes allowances for several factors which affect the performance of organization members in the long run and which had previously been neglected. The resulting theory, it is hoped, describes many of the characteristics that are desirable in task-performing organizations, from command and control systems to business firms. The bulk of the characteristics that still are neglected are those which influence the dynamic time-dependent processes.

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# 1. INTRODUCTION

## 1.1 General Comments

This report presents many of the more recent results obtained from an effort at developing a mathematical organization theory. One can hope that this effort will contribute to an understanding of the function, and malfunctions in systems of men and machines, and later on, perhaps also to an improvement of their performance. In view of the increasing size and complexity of organizations, and of the increasing requirement for high performance, an effort in this direction seems well worthwhile.

By this writing, a considerable body of theory has been developed. It may, therefore, be useful to start here with an overview of the results that have been obtained in this line of work since its inception. This will be done in this section. At the same time, an attempt will be made to compare the theory with others, qualitative as well as quantitative, in order to bring out the parallels and the differences, and with observations of organizations in practice, in order to indicate the extent to which the theory can be validated at present.

Broadly speaking, only non-numerical results have been sought so far, and validation can, therefore, be attempted by a comparison of these results with observations in practice. Such comparisons, however, are thought to be quite encouraging. As the work progressed, the mathematical organization models developed many of the characteristics of their counterparts in practice, starting from some rather simple (and somewhat outmoded) forms to others with features to which one can attach very modern interpretations such as "participative management", "design of work", and "management by exception".

In fact, one can perhaps even discern the outlines of a process by which the theory could be applied to organizations in practice.

## 1.2 Comparison With Other Work in Organization Theory

The present theory was limited at the outset, to certain kinds of organizations, namely those which are set up in order to accept certain inputs (in forms of information, materials, money, etc.) and which convert those into certain outputs of the same or different forms. They are "systems", in the sense in which the term is used in system science. One can then typically define performance criteria for them, which characterize the degree of success achieved in the input/output transformation that is to be executed.

Sociologists who trace the history of their study of organizations over nearly a hundred years back to Max Weber (and in some cases over 2,000 years back to Plato; see, e.g. [33] for a stimulating historical review), often distinguish four types: the mutual benefit organization, the business firm, the service organization, and the commonweal. Surprisingly, only a few include the types which probably are the precursor of at least three of these, namely the military ([2], p. 69). They all fail to consider several other, very modern, organization types which lack many or all sociological ingredients, but which are correspondingly simpler conceptually. These are command-and-control and similar systems, parallel computers, and computer networks.

Very little of the work by sociologists has been quantitative and none has had a goal that was commensurate in breadth with their qualitative studies. In fact, in all of the books and papers on the subject of organization theory that are known to this writer, none even asks a question to which one must have

a clear answer before any mathematical inquiry can begin. This is the question of why organizations exist in the first place.

Quantitative approaches to organization theory have been made, but by economists, rather than by sociologists, and the question regarding the reason for this existence had to be answered by them, as a matter of course. Arrow [2] (p.33), assumes that organizations are means for achieving the benefits of collective action in situations in which the price system fails. Starting from this viewpoint, one can expect a theory to be developed which describes the mutual benefit and commonweal organizations. A second approach which is likely to be more appropriate to the several other types mentioned above, is J. Marschak's Economic Theory of Teams [21]. A team, in this theory, is a system which is charged with an input/output conversion task, and whose performance is characterized by a criterion of the kind mentioned above. It exists because its components, mainly for reasons of economy, are limited in number, can each receive only some of the inputs, and can generate only some of the outputs. It may be roughly correct to say that the development of team theory placed the main emphasis at studying the structure and the decision rules that are induced in such systems by the trade-off between economics within a team on one hand and its performance on the other.

The theory discussed in this report, as well as in an earlier one [10], is concerned with systems of essentially the same kind as team theory. However, it starts from the premise that some of the most striking features of organizations in practice are induced in them, not so much by economic considerations but by the capabilities and limitations of their members. In other words, its basic assumption is that many organizations exist mainly in order

to exploit the capabilities, and to overcome the limitations of their members.

The incentive for this approach was provided by the observation that there are organizations in practice, for instance the military, which presumably are (and, even more so, were) set up without much consideration of economics. Nevertheless, some of the main structural and operational features of an army are, and were, very similar to those of many modern business firms, and many private and public administrative agencies. One can, therefore, perhaps surmise that these features are due to the fact that the components of all of these systems are human beings, and hence, that some of the strengths and limitations of man are responsible for their most striking characteristics.

This premise strongly affected the work on this theory from the outset. Since the objects of the study were to be systems which had human beings as their components, a considerable effort had to be devoted to the mathematical modelling of what might be called the "organization man". Any such model can , of course, only be a great simplification, but the problem was, and still is, what particular traits are needed to account for observed organizational features. This effort is summarized below in Sect. 1.3. This line of work had to be supplemented by second and larger ones, namely the derivation of the characteristics of an organization from those of its members. It is reviewed in Sect.'s 1.4 to 1.7. The main objective there was, and still is, the derivation of results that could be compared with observations on organizations in practice. Only in this way, it was felt, one could hope for a validation of the theory and, ultimately, for its application in the field.

The work in both directions, i.e. towards an adequate model of the human organization member and towards a verifiable model of organizations, pro-

ceeded through several stages of refinement (not to mention several stages of failure). Several additional ones are already discernible now. One of these is a fusion with team theory. The pre-occupation of the theory with the interplay between the characteristics of a system and those of its components, to the exclusion of economic considerations, is patently indefensible in the long run.

### 1.3 Characteristics of Organization Members

As mentioned above, the present theory deals with organizations whose primary task is the transformation of certain inputs into appropriate outputs. Its main premise is that the characteristics of such organizations are directly or indirectly determined by those of their members. It is thus necessary to specify the latter.

A typical organization member is assumed to be capable of executing the kind of task which the organization as a whole is to perform, i.e., the conversion of an input into an output. On the other hand, he is also assumed to be limited, roughly speaking, in the quantity and quality of the conversions which he can perform in a given period of time. In the currently popular terminology, one would probably say that he is limited in his productivity. Evidently, this kind of statement must be made more specific. In fact, it is desirable to do this in a way which does not necessarily apply only to the human members of an organization but also to the machines which are included in many modern systems and which are the exclusive components of some.

The way in which the quantity of a member's throughput was defined is explained in detail in Sect. 3 of [10]. Briefly, every member is assumed capable of acquiring and interpreting his inputs in certain units, called "input

symbols" $x_i$, and of converting them into certain "output symbols" $y_j$. He is further assumed to require a certain "processing time" $t_{ij}$ for each such conversion. His mean processing time, in effect, determined the quantity of effort of which he is capable.

The introduction of processing times, as a characteristic of member's work capacity, establishes direct connections between this theory and several others. He can be considered a communication channel in the terminology of information theory ( [27], p. 7). He can also be considered a server in a queuing system, and a number of writers (e.g., [18], and [6], p.389) have, in fact, taken this view.

The quality, by contrast to the quantity, of the performance of an individual organization member is measured in this theory relative to a standard which is assumed prescribed for him. It is, in other words, assumed that certain input/output conversion procedures are laid down for him as being desirable, and any departures from them are considered undesirable. The quality of his effort is then related to the probabilities of such departures, as described in detail in Sect. 3 of this report. Some departures are considered unavoidable and written off as typical human (or machine) fallibility. An excessively high incidence, however, calls for correction.

A main cause of excessive incidence is assumed to be overload. Psychophysical evidence, as well as common sense, confirm the existence of such a concept. Overload can be due to the fact that the rate of arrival of the input symbols, or that the rate of the required output production, or both, are too high. The first has received sporadic attention in the literature, under the designation of "input" or "information" overload. The latter seems to be

too obvious and wide-spread a phenomenon to have earned for itself the obvious term of "output" overload.

Overload, in either sense, has not been considered an important concern in organization design, to the writer's knowledge. In this theory, it plays a crucial role. In fact, one of the main conclusions of it is that the most striking features of organizations are introduced by the requirement that overload be avoided throughout, if at all possible. The reason for this is probably that the incidence of errors of all kinds is low enough in human beings to be acceptable for most situations but that overload quickly raises it to unacceptable levels.

If one assumes, as has been done here, that the part a member plays in the execution of the tasks of an organization is characterized primarily by the quality and quantity of his work, then it follows further that any other of his traits play a secondary role: they matter only to the extent to which they affect the primary ones. Such secondary factors can nevertheless be quite important. In fact, without them, at least some of the features observed in organizations in practice would not even exist. Accordingly, considerable effort was expended in this study on the question of what secondary factors are likely to be the most significant to a person's performance in an organization, and how they should be included in the theory. No comparable effort has been expended on the identification of such factors in machines or, for that matter, on the factors that are introduced by the co-existence of men and machines in many systems.

In pursuing this line of inquiry, a natural division developed between the secondary characteristics of the members in two kinds of organizations. One

is the type designed to execute short-duration, high-demand tasks, such as command-and-control systems. The relevant variables in such cases have been identified, thanks to psychophysical experimentation (see, e.g. [15]). They are:

(i) the number of the input symbols for which the member must be prepared;

(ii) the probabilities with which these symbols arrive;

(iii) the number of destinations for the output symbols; and perhaps also

(iv) the probabilities with which these symbols are distributed among their destinations.

These factors were called "load factors" in this study, in a somewhat debatable terminology (because they characterize, in a way, the difficulty of a given input/output conversion). A member whose productivity depended on some or all of them, is "load-dependent". The details of load-dependence are developed in Sect.'s 3 of [10] and of this report.

In organizations, such as business firms and administrative units, whose tasks are typically of long duration and represent a fairly steady load, the productivity of a member depends on certain motivational factors that have been identified by social psychologists. The identification is not as unique as that of the load factors (one can characterize the dependence by many different sets of factors) nor is the cause/effect relationship as well documented. Those that have been found most appropriate to this study are reported by Porter, et. al. [26]. They are:

(a) the degree of autonomy under which an organization member works;

(b) the status which he feels he occupies vis-a-vis the organization and his peers;

(c) the extent to which he is satisfied by his job.

They are called " $\psi$-factors" in this report, using some terminology that is debatable. A member whose productivity is affected by them is " $\psi$-dependent". The line of reasoning that was used to reduce these concepts into mathematically well-defined variables is presented in Sect. 5 of this report.

The model of the generic organization man that emerges from these considerations for either organization type may be oversimplified but it is quite complex, by the standards of systems work. The study of systems, therefore, which are made up of such complex components is a rather challenging undertaking.

### 1.4 Organizational Design

The "design" of an organization is often interpreted as the drawing up of an organization chart. This is then called the "structure" of the organization. In this theory, the specification of the structure is only part of the job of the "Designer". Moreover, the chart that he prepares is not a conventional organization chart. The role given to the Designer was quite important, at least in the initial portion of this study. He was assumed to be completely informed regarding the nature of the task, as well as regarding the characteristics of each prospective member of the organization. This was a very convenient assumption to start with, but it had to be gradually weakened as the study progressed, in a way that will be explained below.

An organization chart, in mathematical terminology, is a directed graph (or digraph) whose nodes are the organization members. In the conventional

chart, the arcs are said to be representative "lines of authority". It is not quite clear what is meant by that, or for that matter, to what extent an organization is specified by these lines. Misunderstandings are avoided if the arcs of the chart represent the official communications links of the organization. Deutsch [9] has suggested this representation as advantageous also from the sociologist's viewpoint. J. Marschak [21] uses it in team theory almost as a matter of course. The actions of the organization member can be interpreted in several ways, most conveniently perhaps, as decision-making. This has been suggested by Simon [30].

In this theory, the organization chart is interpreted as a specification of the official transmission links between members. It has the advantage that one can imagine the arc of this graph labelled with the "symbols" that are transmitted over them, and if possible, also with the probabilities that govern those transmissions. For it develops that a complete specification of an organization in priciple requires such labels. They define the operating procedures which the members should follow in order to execute their joint task. The collection of these labels is called the "standard operating procedure" or "SOP" of the organization (although the term "standard" may not always be appropriate, as will be explained in Sect.'s 1.6 and 1.7).

The designer of an organization, therefore, must in principle, define its structure, in terms of its communications net, as well as its SOP. He must choose them in such a way that the organization performs its task as well as possible, despite the limitations in the quality and quantity of the work of which its members are capable.

It develops that these limitations virtually imply some coarse structural features and that the Designer will adopt them almost as a matter of course. He will find, first of all, that the organization divides rather naturally into two divisions. The first executes the input/output conversion which is the primary task of the organization and the other monitors the performance of the first. They have been named the "executive" and the "control" branches.

In the executive branch, he will furthermore often be able to distinguish two-subdivisions, namely the "staff" and the "line". The second term is used in the theory in very much the same sense as in practice. It designates the segment of the organization which is charged with the production of the outputs. The term "staff", however, is given a somewhat more general sense than the one that is usually attached to it. It applies to those members whose job it is to process the organizational inputs.

In practice, the distinction between the executive and the control branches, and between the staff and line, are rarely as neat as it is in the theory. Members of the executive branch often act also as controllers, and members of the line often carry out staff functions. In fact, the distinction made by the theory should be considered functional, rather than personal.

Organization theory in the past has been concerned mostly with the line. The staff function is often discussed, but never very satisfactorily in this writer's opinion. The same applies to the control branch. The fact that the mathematical theory described here led very quickly to some rather definite interpretations of the function of all three organizational divisions, may indicate that its basic premises are rather close to the mark. These indications

are further reinforced by the results to be described next.

## 1.5 Centralized Organizations

The immediate further development of the theory was made subject to a restriction which was expected to simplify it, and which did in fact do that to a gratifying extent. This was a restriction to organizations which are strictly "centralized", in a certain well-defined sense. It is that one of the members called the Executive (or Exec, for short) has a distinguished position: all inputs to the organization are required to be channelled to him, in one form or another, before they are transformed into the appropriate outputs.

The simplification due to this restriction was a considerable advantage in the first phase of the work, most of which has already been reported on in [10]. The results obtained from it added further support to the basic premise of the theory, namely, that the limited capabilities of the members are the main determinants of the structure and operating procedures of many organizations.

A line, for instance, which is made up of load-dependent members is always best hierarchical, in the way it is presented in innumerable organization charts. The "lines of authority", in other words, coincide with the official communication links in these cases. One can, in fact, prove also a kind of converse to this statement. If the line unit of a centralized system of any kind performs best with a hierarchical structure, then it is the avoidance of overload, or of some similar saturation phenomenon, which induces that structure. It further develops that the line can operate in two and only two modes (or their combinations) which were called "parallel" and "alternate" processing. Conditions were derived under which overload can be avoided under either mode, and characterizations were

obtained for the SOP's.

The staff was found to differ from the line structurally and operationally. Its structure may be hierarchical but it may also be more amorphous. Its operations then rather resemble that of a committee or, in sociological terminology, of a "group". Combinations of hierarchical and group structures are possible also. The function of the staff in a centralized organization is the pre-processing of the organization inputs in a way which avoids overload on the Executive. It often achieves this by a type of operating procedure which has been called "equivocation", a term borrowed from information theory. It is more general than the process of aggregation (or compaction) which has often been recognized as a feature of organizational pre-processing.

Only results of a non-numerical nature were sought because of the scarcity of reliable numerical data. Those that described structural properties of organizations were, it was felt, in good agreement with observations in practice. Very little is known regarding the general properties of SOP's used in practice, except perhaps that stochastic ones are avoided, if possible. The theory shows that the best SOP's indeed usually are non-stochastic.

The simplifications due to the restriction to centralized organization were, however, purchased at some expense. In particular, they introduced some loss of realism (and in fact more of a loss than was realized initially). The mathematical models that were obtained resembled the organization forms that have gone out of fashion several decades ago. Actually, a very few, if any, organizations were strictly centralized even then. The mathematical models, in other words, resembled the abstraction of an organization that was considered to be well-conceived some time ago, but rarely was the real thing.

## 1.6 Decentralization

In a decentralized organization, the Executive does not have the special position in the organization chart that has been described above, namely that of being a center through which all inputs must be channelled. Rather, transmission links are used to by-pass him. Some of these give the line direct access to the staff, and others to sources completely outside the organization. The latter seem to be more interesting and important.

In fact, it seems to be an essential characteristic of a decentralized organization that it permits some of its members to utilize inputs from sources that are either altogether inaccessible to others, or else that are accessible only with considerable difficulty or expense. Such sources are called "private" in this study. As the term implies, they may be the origins of some special intelligence which is available only to one member. More generally, however, they may represent merely a member's familiarity with his immediate environment, or his personal experience and judgement, or some body of information on which only he can draw.

Decentralization is the main topic of this report. Sect.'s 4 to 6 deal with it. Sect.'s 4.2 and 4.3 first show that it leads to performance improvement in many cases. The reason for this is easily explained. In a centralized organization, the Executive represents a rather formidable bottleneck, the effect of which even the best of equivocations by the staff can only mitigate. The addition of transmission links that by-pass him is typically much more profitable. The modern trend towards decentralization in many organizations implicitly exploits this effect.

It exploits another, and more subtle, one as well. This is the motivation towards higher productivity with which a member responds to the greater autonomy he enjoys under a decentralized arrangement. Autonomy, it may be recalled, was introduced as one of three motivational effects which were called $\psi$-factors in Sect. 1.3. It also appears to be the most important of the three because the structure and operating procedure of an organization are more strongly influenced by it than by the others. This is explained in Sect. 6.2 of this report. In fact, since the degree of a member's autonomy is related to the latitude he has in the choice of his operating procedures, the looseness in their specification is a characteristic of many decentralized characteristics.

Another characteristic which is also often observed is a change in the nature of the line hierarchy. In centralized organizations, it is of course strongly oriented towards the Executive while in decentralized ones there is a tendency to the formation of groups, each consisting of a supervisor and his immediate subordinates and operating very much like the staff groups mentioned above. One can perhaps interpret this tendency as one towards "participative management" which is being advocated by some behavioral scientists.

The performance improvement that one can often expect to accrue from decentraliztion may not materialize, however, because there are several adverse effects which tend to cancel its benefits. One is an increase of input load among line and, with it, the risk of overload. More important, however, is a phenomenon which is an intuitively obvious consequence of decentralization. This is a partial loss of coordination. It develops for several reasons, all of which could be avoided by careful design, at least in principle, if only the processes with-

in the organization were perfectly predictable. The fact, however, is that they are not. For one, its members are error-prone, in the sense that they will fail to execute an SOP perfectly, no matter how simple it is or how carefully it has been laid out for them.

Most members of an organization, however, notably those on supervisory levels, have no precise SOP which they can follow. This failure to provide them with one may be by design: a fixed SOP would wipe out their autonomy and, hence, the motivational performance improvements that go with it. More often, however, the failure will be due to the fact that that the kind of organizational design which one can perhaps postulate in principle, is never possible in practice. The Designer in other words is never as all-knowing and infallible as he was assumed to be in the early stages of this study. Loss of coordination in an organization, and the degradation of performance that may result from it, will thus frequently be due to faulty or incomplete design.

1.7 Organizational Control

The purpose of control in any system is, broadly speaking, the avoidance of performance degradation. In organizations, control should more specifically detect and correct departures from the desired performance.

It had been hoped in this study that a portion of the considerable existing body of control theory could be brought to bear on the problem of organizational control. Stochastic optimal control theory in particular seemed as most promising candidate. These hopes remained largely unfulfilled. In fact, the problem required a number of novel insights into the control process in general, partly because organizations in practice can execute it in many different and unusual

ways, and partly because their internal functioning often is largely unspecified, as has just been explained. As a consequence, the study of organizational control proliferated into several investigations, each proceeding along fairly distinct lines.

One was the investigation of "error control". An error, in the terminology used in this study, is any departure by a member from an SOP that is prescribed for him. This type of control, therefore, assumes that such a prescription exists in the first place, which will be true only for members who have virtually no autonomy. Whenever it can be used at all, it can be (and, in practice, is) executed in two ways, namely by feed-forward or by feedback control. The first can be dealt with in the same way as diversity reception in communications engineering. The second can be treated by feedback control theory, however, only in those cases in which the correct SOP is not only known, but non-probabilistic as well.

However, many SOP's in the theory, and some in practice, are stochastic. Most of feedback control theory even in its most modern versions then becomes ill-suited, or worse, for the treatment of the problem. The approach that seems most appropriate, intuitively and mathematically, is a very recent new-comer to the field, namely the theory of "error-tolerant" control [11]. This approach is, in fact, appropriate also in situations when an SOP is not fully specified or, for that matter, when the characteristics of the organizational task, or of the organization members, are incompletely known. A feature that recommends this control principle is that it is partially adaptive. Another, and related, feature is that corrective action is taken only if the departure from the desired performance, and perhaps also its cause, have clearly exceeded a known level of toler-

ance (hence the epithet "error-tolerant").

Corrective action can then be taken by "equivocation" or by "intervention". In the first case, the controller functions very much like a staff member. He intercepts the input to the controlled organizational unit and modifies it in a way that is expected to improve performance. In doing so, he must take into account that many of these modifications infringe on the autonomy of the controlled unit, and hence, may be counterproductive for that reason.

Control by intervention, on the other hand, induces changes in the procedures which are followed by the controlled unit, in the motivational factors of its members, and possibly also in its structure and composition. Some of these changes are especially appropriate in organizations which operate in a variable environment and in which considerable adaptability is required. The control branch in such instances may have to be quite carefully designed. It may then have its own staff and line divisions, the first for the collection and evaluation of performance data and the second for the selection and execution of the appropriate corrective action (if any). It will often have its own Executive, whose position may be quite important. If the organization's success is strongly tied to its adaptability, his may be the most important position of the organization, and it would then presumably be occupied by its most effective and experienced member. He would, in any case, take action only if there were indications of a drop in the desired performance of the executive branch which exceeds the tolerance level. He would, in today's terminology, "manage by exception" or "command by negation".

In organizations in practice, as has already been mentioned, the separation between the executive and the control branches is not as clean as in the theory.

Much of the control function tends to be executed by the members of the Executive branch. Some of this may be traceable to the need to preserve autonomy, but the rest may be just poor practice. Reports of poor organizational control practices certainly abound in the sociological literature (see, e.g., [26], p. 265), and improvements in this area seem quite badly needed.

## 2. ORGANIZATIONAL GOALS

The theory that forms the subject of this report is to lead, by way of a mathematical line of reasoning, from certain assumptions regarding the members of an organization to the structure and operating procedures which the organization should adopt in order to execute its task. In fact, one can require that the execution should be in the best possible way. In either case, assumptions must be made regarding the nature of the task. The purpose of this section is to set down those that have been made in the theory so far.

The assumptions should be chosen as simple as possible, yet in such a way that their simplicity doesn't eliminate the desired realism from the results that are to be derived for the organizations. A set that satisfies these requirements defines a decision-making task of a kind which has also been used by J. Marschak [20] as a basis for his Theory of Teams. It was described in Sect. 2 of [10], and it is re-stated here, for sake of completeness. In brief, it is this.

It is assumed that the organization receives an input drawn from a certain set of inputs and is required to respond to it with an output from a similar set. It is rewarded for such a response with a certain pay-off which in general depends on the response, as well as the input. The goal of the organization is to maximize the mean pay-off.

In order to make this definition more specific, it is convenient to introduce some terminology, much of it borrowed from information theory. Thus, it will be useful to consider the inputs to come from "sources" and the outputs to be delivered to "destinations", both outside the organization. It will further-

more be convenient to speak of all inputs and outputs as being "signals", regardless of whether they actually are communication signals. The same term will also be used for the transmissions among the organization members. In other words, any kind of transmission of information, material, money, etc. involving the organization or any of its members will be subsumed under the term "signal".

A signal will be understood to be a sequence of certain units which will be called "symbols" here, however, with the understanding that this term should also be interpreted quite broadly. A symbol may be a single letter, or a sentence, or a whole memorandum. It may also represent a number of dollars, or a batch of material, or a product of manufacture. The collection of different symbols that are used in a signal are its "alphabet". All alphabets in this study are assumed to be finite (but possibly quite large).

The members of the organization will be assumed to operate in the same way as the organization as a whole. They will, in other words, acquire symbols as inputs, either from outside sources or from other members, and will dispatch them after suitable processing to certain destinations inside or outside of the organization. The members can be thought of as persons or machines. For convenience, however, they will be spoken of as persons and referred to as "he" or "she". In fact, rather than designate them with impersonal labels such as letters or numbers they will be referred to the letters of the NATO alphabet, i.e., with Alpha, Bravo, Charlie, Delta, etc. The Executive, or Exec, who was already introduced in Sect. 1.5, will always be a member of the organization, also.

Suppose now that $x_j$ is an input symbol received by the organization at a certain point of time, either from one source or jointly from several. It is

assumed that $x_j$ is acquired with a known "symbol probability" $p_j$, and that one such symbol is acquired per unit time. The incidence of symbols in different time units is assumed to be statistically independent. This independence is assumed to be preserved throughout the organization and in the end imparted to the outputs.

The organization is required to respond to the input symbol $x_j$ with an output symbol $y_k$, and to deliver that to one or more destinations, It need not do so within the same interval as the acquisition of $x_j$. On the contrary, delays are permitted, provided only they are not cumulative. That is, the average delay between the receipt of an input symbol and the delivery of an output symbol must not exceed one time unit. The organization, in other words, must be able to keep up with an average schedule of one output symbol per unit time.

If it responds to $x_j$ with $y_k$ it receives a reward $L(j;k)$. It is of course desirable for it to respond to each $x_j$ with $y_k$ that maximizes the reward. If it can actually do that, it surely performs optimally. However, under the assumptions to be made here concerning the individual organization members, especially in situations in which they have difficulties in keeping pace with the schedule, this optimal response will not always be realized. Rather, when the input symbol $x_j$ is received, every output symbol $y_k$ is a possible response and in fact will be the response with a certain probability $p(y_k | x_j)$. The performance of the organization will then be rated by the mean pay-off which it manages to collect, i.e., the quantity

$$(2.1) \qquad E\{L\} = \sum_j p_j \sum_k L(j;k)\, p(y_k | x_j) \; ,$$

in which j and k range over the input and output alphabets, respectively. The greater this quantity, the better the performance. Optimal performance is of course a special case of (2.1): if the largest pay-off that the organization can reap for a response to $x_j$ occurs when $k = k(j)$, then $E\{L\}$ will patently be maximized by

$$p(y_k \mid x_j) = \begin{cases} 1 & \text{for } k = k(j) \\ 0 & \text{otherwise.} \end{cases} \tag{2.2}$$

This therefore is the decision-rule which the organization should realize, if it can.

On several occasions, an input symbol $x_j$ will, in fact, be a pair of two symbols, $(u_i, v_j)$ for instance. In such cases, $L(i, j;k)$ will be used in place of $L(j;k)$. An analogous notation will indicate that an output symbol represents a pair.

This completes the definition of the organizational goal. It is seen to be a decision-making problem of quite a conventional kind, except possibly for the requirement that the organization must stay on schedule.

The main goals of organizations in practice are often more complicated in several respects, and should be modelled accordingly. One unrealistic assumption above is that the temporal statistical independence of all processed symbols. Another is that there is neither a penalty nor a reward for processing delays. According to present indications, however, the results obtained so far are unlikely to be greatly affected by changes in these assumptions.

A third assumption, however, which is implicit in this problem formulation does have a rather significant effect. This is the assumption that the quanti-

ties on the right-hand side of (2.1) are known. In practice, this is hardly ever the case. Organizations apparently deal with this problem by adapting their procedures in accordance with what they do know, and change them if a change seems indicated. The present theory deals with the problem in the same way, as was described briefly in Sect. 1.7.

## 3. THE "LOAD-DEPENDENT" ORGANIZATION MEMBERS

### 3.1 Load Dependence

The view was taken in [10] that many characteristic features of an organization, among them in fact some of the most striking ones, are direct or indirect consequences of those of its members. Organizations, in this view, are more specifically designed to utilize the strengths of their members, and to minimize the effect of their limitations. Evidently, in order to exploit this idea, assumptions must be made regarding the pertinent properties of the members. This was done in Sect. 3 of [10], but only up to a point.

The main strength of a member was assumed to be his ability to execute tasks of the kind which the organization as a whole is supposed to perform, i.e. the transformation of inputs into outputs. His limitation lay, roughly speaking, in the speed and in the accuracy with which he was able to carry out his task. The speed of his performance was more specifically taken to be determined by his "processing times", i.e. the times he needed to acquire and interpret his inputs plus those for the production and delivery of his outputs. (The distinction between separate input and output processing times made in [10] will not be retained here. It has not led to the expected conceptual simplification.) The accuracy with which he performed his task was largely disregarded in [10], by an explicit assumption. It is however considered in the work reported here, in a way that is explained in the sections that follow.

Both the speed and the accuracy of task execution in turn depend on many factors, especially if the member is human. However, for the purpose of the present study only those matter which are consequences of his task and his position within the organization. They seem to fall rather naturally into two classes: those which characterize the difficulty of the task, at least for the member who is to execute it, and those which represent the effect on his performance of the structure and the operating procedures of the organization. The first will be called "load factors" here, and their

effect on his performance as "load dependence". The second will be referred to as " $\psi$-factors" and their effect " $\psi$- dependence".

Load dependence of the speed with which an organization member executes his task (or, a bit more precisely, of his processing times) was considered in [10], largely because there is fairly incontrovertible evidence of its effect from experimental psychophysical data. Load dependence of the accuracy of task execution is discussed in the next sections. The $\psi$-dependence of speed as well as accuracy is the subject of Sect. 5.

### 3.2 Fallibility Among Organization Members

One of the assumptions that underlay the work reported in [10] was that, unless he is overloaded, every organization member functions in the way prescribed for him by his standard operating procedure (SOP). Thus, if his SOP is pure and requires him to respond to an input symbol $u_i$ with an output symbol $v_i$, he will unfailingly do so unless he is overloaded. Similarly, if his SOP is mixed and prescribes for him a set of transition probabilities $p^*(v_i | u_i)$ with which he is to choose among the possible outputs $v_k$, then he will actually make his choices accordingly. In practice, this does not happen. Organization members, man as well as machine, are error-prone to some extent and cannot execute any SOP precisely as prescribed. Quite often this is due to the fact that their inputs are contaminated with noise from some external source but even when they are not, a residue always remains, in humans due to fallibility or poor judgment and in machines due to internal noise or inherent imprecision.

It will be convenient in what follows to distinguish two kinds of departure from a prescribed SOP. One will be called an "error". It occurs in those situations in which an organization member is expected to follow a pure SOP but fails to do so. He will then occasionally respond to an input symbol $u_i$ with an incorrect output symbol $v_k$ instead of the $v_i$ which is prescribed for him. It will be assumed here that he does so with the "error probability" $p(v_k | u_i)$.

The second kind of deviation will be called a "malpractice". This term will be used in the more general situations in which the prescribed SOP is mixed or inadequately specified, and in which he realizes certain transition probabilities $p(v_k \mid u_i)$ instead of the desired $p^*(v_k \mid u_i)$. It will not matter, in what follows, whether errors or malpractices are due to external noise or internal imprecision but it will be convenient to speak of them as being caused internally. A member will be called "error-prone" in either case, and "error-free" otherwise.

3.3 Load Dependence of Errors

As mentioned in Sect. 3.1, there is fairly conclusive experimental evidence of the phenomena that are referred to as load dependence here in the processing times of human organization members. There is evidence also of such dependence in the frequency of their errors but it is not as well, and not as usefully, documented.

Processing times are known to depend on the range $n$ over which the input symbols $u_i (i = 1, 2, \ldots, n)$ may vary, as well as the range $m$ of the output symbols $v_k$ $(k = 1, 2, \ldots, m)$. Thus, if $t_{ik}$ is the processing time for the transformation of $u_i$ to $v_k$,

$$t_{ik} = t_{ik}(n, m).$$

In fact, the $t_{ik}$ are monotone non-decreasing in n as well as m. It is further known that the $t_{ik}$ also depend on the probabilities $p(v_i)$ with which the input symbols are received, and one can surmise that they similarly depend on the probabilities $p(v_k)$ with which the output symbols are dispatched to their destinations. These dependences, at any rate, were allowed for in the $t_{ik}$.

The error probabilities $p(v_k \mid u_i)$ are known to depend on n, the size of the input alphabet [15] but little is known regarding this variation that is reasonably quantitative and unequivocal. One can deduce that, as n increases, the probability of the correct responses decreases. The probabilities

of erroneous responses correspondingly tend to increase, with the result that the response distribution loses some of its "peakedness". The trend, in other words, is towards a greater entropy among the responses $v_k$ to a given input symbol $u_i$. Data regarding the effect of increasing the size $m$ of the output alphabet seem even scarcer. Nevertheless, one can surmise that the same trend prevails also in this case. An increase in $m$ induces an increase in the entropy among the $v_k$ for each $u$.

These comments lend more specifically to the following assumptions. Let $H(v|u_i; n, m)$ be the entropy among the responses $v_k$ to the input $u_i$ when $n$ and $m$ are the sizes of the input and output alphabets. That is, for $i=1,2,\ldots,n$, and $k = 1,2,\ldots,m$,

$$H(v|u_i;n,m) = -\Sigma_k\, p(v_k|u_i;n,m)\log p(v_k|u_i;n,m) \tag{3.1}$$

The assumption then is that, for $i = 1,2,\ldots,n-1$,

$$H(v|u_i;n,m) \geq H(v|u_i;n-1,m) \tag{3.2a}$$

$$H(v|u_i;n,m) \geq H(v|u_i;n,m-1) \tag{3.2b}$$

Eq. (3.2 a) does not specify what happens for $i=n$, i.e. for the error probabilities induced by a newly added input symbol $u_n$. One can perhaps assume that entropy of the responses to $u_n$ is no less than those of the responses to $u_i$, $i=1,2,\ldots,n-1$. I.e.,

$$H(v|u_n;n,m) \geq H(v|u_i;n,m) \qquad i=1,2,\ldots,n-1. \tag{3.2c}$$

This will in fact be assumed here, along with (3.2a) and (3.2b).

One can show that assumptions (3.2) are fulfilled by all mathematical models of error incidence that have, to the writer's knowledge, been considered in the literature [20]. One can also readily prove the following.

Proposition 3.1. Under the assumptions (3.2), an increase in the size of either the input or output alphabet, or both, increases the equivocation

$$H_u(v;n,m) = -\Sigma_i\, p(u_i,n)\, H(v|u_i;n,m) \tag{3.3}$$

or else leaves it invariant.

Proof. Consider first an increase in m, the size of the output alphabet. Omitting n in (3.3) for convenience,

$$H_u(v;m) - H_u(v;m-1) = \Sigma_i\, p(u_i;m)\,[H(v\,|\,u_i;m) - H(v\,|\,u_i;m-1)] \geq 0$$

by (3.2b). Similarly for n and with m omitted,

$$\begin{aligned} H_u(v;n) - H_u(v;n-1) &= \\ &= \Sigma_i^{n-1}\,[p(u_i;n)\,H(v\,|\,u_i;n) - p(u_i;n-1)\,H(v\,|\,u_i;n-1)] + \\ &\quad + p(u_n;n)\,H(v\,|\,u_n;n) \\ &= \Sigma_i^{n-1}\,p(u_i;n)[H(v\,|\,u_i;n) - H(v\,|\,u_i;n-1)] + \\ &\quad + \Sigma_i^{n-1}\,[p(u_i;n) - p(u_i;n-1)]\,H(v\,|\,u_i;n-1) + \\ &\quad + p(u_n;n)\,H(v\,|\,u_n;n) \end{aligned}$$

But

$$p(u_n;n) = 1 - \Sigma_i^{n-1}\,p(u_i;n) = -\,\Sigma_i^{n-1}\,[p(u_i;n) - p(u_i;n-1)]$$

so that

$$\begin{aligned} H(v\,|\,u;n) - H(v\,|\,u;n-1) = &\ \Sigma_i^{n-1}\,p(u_i;n)[H(v\,|\,u_i;n) - H(v\,|\,u_i;n-1)] \\ &- \Sigma_i^{n-1}\,[p(u_i;n) - p(u_i;n-1)][H(v\,|\,u_n;n] - \\ &- H(v\,|\,u_i;n-1)] \geq 0 \end{aligned}$$

because, by (3.2c), the terms of the first sum above are no smaller than their counterparts in the second. This is as was to be proven.

One might consider extending the parallel between the load dependence of the processing times $t_{ik}$ and that of the error probabilities $p(v_k\,|\,u_i)$. In such a case, one would have to allow the $p(v_k\,|\,u_i)$ to be functions of the input and ouput probabilities $p(u_i)$ and $p(v_k)$. The entropies

H in (3.1) and (3.3) would then have to be replaced by functions of a more general form. Requiring them to be concave, with a maximum when all $p(u_i)$ and all $p(v_k)$ are equal, seems a natural generalization. Available data however are too scanty to suggest such a step, and it therefore will not be taken here.

There is however another parameter of load dependence which does enter the error probabilities and which must be at least mentioned here. This is the required mean processing time $\tau$. It is known from experimentation [25] as well as observation (e.g., [7]) that the frequency of errors increases strikingly when overload develops in an organization member, i.e., when $\tau$ is larger than the average time available to him for the execution of his task. The required mean processing time $\tau$ should therefore be entered along with m and n as a parameter into the error probabilities $p(v_k \mid u_i)$ and, through them, into the entropies in (3.1) and (3.3).

In the analysis to be presented in this report, however, the dependence of $\tau$ will be avoided by the same device as in [10]. It will be assumed that the design of the organization is so drawn up that overload is avoided throughout. One of the main points of the theory presented in [10] was the demonstration that that can be achieved in general and that in fact most organizations in practice are designed to achieve it. The assumption accordingly is reasonably well justified, at least at the present stage of the theoretical work.

## 3.4 Load Dependence of Malpractices

A malpractice, in the sense which was attached to the term in Sect. 3.2, is a departure from an SOP $p^*(v_k \mid u_i)$ which is desired of a member. It may occur in those instances in which the desired SOP is known to the member, but in which he is incapable of putting it into practice in precisely the way it is prescribed for him. More often however it will develop in situations in which $p^*(v_k \mid u_i)$ has not been completely specified to him and in which he fails to realize that this is the SOP which is best for his position. Such situations are dealt with in Sections 5 and 7 of this report.

Malpractices of these kinds are well known facts of life. However, the writer has so far found no indication in the literature that data have been collected on their statistical characteristics or, for that matter, that their effect on organizational functions and design have even been considered by organization theorists. Under the circumstances, only some rather broad qualitative assumptions can be made here regarding their load dependence.

It will more specifically be assumed that the conditional entropy (3.1) of a malpractice is always larger than its counterpart

$$H^*(v \mid u_i, n, m) = - \sum_k^m p^*(v_k / u_i, n, m) \log p^*(v_k / u_i, m, n) \tag{3.4}$$

for the desired SOP, that both are non-decreasing in n and m as specified by (3.2), and that the increase in H is no smaller than that in $H^*$.

For terminological convenience, H will be called the "error entropy", regardless of whether it indicates the incidence of errors or of malpractice.

## 4. DECENTRALIZATION

### 4.1 General Comments

The question of what is meant by "de-centralization" in an organization, under what circumstances to introduce it, and how far to carry it, has been considered by many writers and from several points of view. That of a sociologist is explained by Fesler [12] who uses the term as roughly synonymous with any delegation or "dispersion" of authority. He traces the history of the phenomenon in chiefly the practice of governmental administration. Carzo and Yanouzas [6 ] (pp. 56-68) base their more recent discussion of the subject on a similar interpretation of the term. Decentralization has also been of interest to economists, as T. Marschak [24] explains in his review of planned economies, namely in connection with the most efficient distribution of resources from a central to certain satellite agencies. The subject has of course been of interest also in management science. However, only one study by Kochen and Deutsch [18] is known to this writer which treats the subject quantitatively. Decentralization has become important recently also in the theory of large engineering systems. Computer networks in particular represent a field in which the topic has become important [ 4 ]. The control of physically dispersed systems, often called "large-scale systems", is another [16]. A large volume of quantitative work has been done in both fields, without however a clear definition of what is meant by a decentralized system.

In this report, the term "decentralization" will be used in a very definite sense, namely as the antonym of "centralization" as defined in [10] (p. 36). According to that definition, a centralized organization has a graph with a cut point (the "Executive" or "Exec") which separates all sources from all destinations or, which is saying essentially the same thing, which separates the staff and the line. A decentralized organization contains no such cut point.

The basic idea is illustrated in Fig. 4.1. It shows the structure of a very simple organization in which the solid lines indicate the transmission

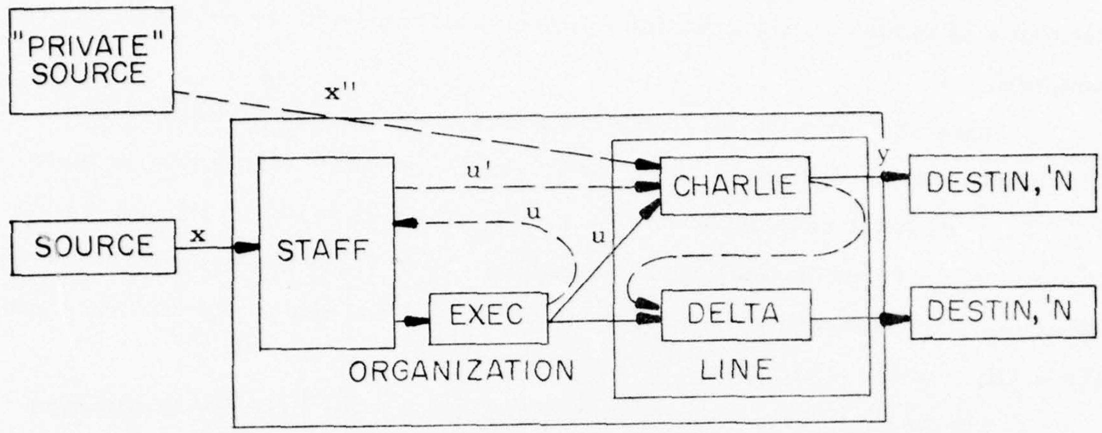


Fig. 4.1

Decentralization in a Simple Organization

links that might connect its members if it were centralized. The dashed lines are typical examples of decentralization. They indicate that line members are given access to sources other than their immediate superiors, as for example Charlie in the figure, who receives some inputs (u') directly from the staff, and others (x'') from a "private" source of his own. As will become clear below, the addition of the transmission links that by-pass the Executive ("by-pass links", in the terminology to be used below) often entail the addition of others, such as the feedback of u from the Executive to his staff, and that of cross links among the line, as indicated in the figure by the one going from Charlie to Delta.

The term "private source" will be given a broader interpretation in this study than that normally attached to it, which presumably is that of source supplying some kind of intelligence to one particular organization member but not to the others. It will be more generally applied to sources of any kind of input which is very readily or quite economically available to one member but not to others. Such inputs may come from the member's familiarity with his personal environment. They may also be derived from his personal experience

and judgment. The notion of a private source, in this broad sense, seems to be quite essential to that of decentralization. In fact, many of the complexities of modern organizations appear to be traceable to it in one way or another.

This section will deal with questions such as these. Under what circumstances is decentralization desirable? How far should one go with it? That is, what restrictions, if any, should be imposed on the use by the line of inputs from the staff and even more so, of those from private sources? What standard operating procedure (SOP) should one prescribe for the Exec, the staff, and the line, under decentralization?

The assumptions under which these issues are to be dealt with are essentially the same as those in [10]. Thus, it will be assumed that structure, as well as SOP, of the organization are drawn up by a "Designer" who is completely and accurately informed regarding the reward matrix that governs the performance of the organization, and regarding the statistics of all inputs, private or not. The properties of the latter are the same as those described in Sect. 2 above, and those of the organization members essentially the same as the ones discussed in Sect. 3 of [10], and of Sect. 3 of this report. One generalization however will be allowed here on several occasions. The assumption will then more specifically be made that organization members may be error prone even in the absence of overload, as opposed to [10] in which this possibility was ignored. The details regarding this last assumption were discussed in Sect. 3 of this report.

The section is organized as follows. Sect. 4.2 discusses conditions under which by-pass links from the staff directly to the line do, or do not, lead to better performance. The assumption is made that all members are error free. Sect. 4.3 removes this assumption. In Sect. 4.4 the range of organizational options is broadened and by-pass links from private sources are admitted. However, cross links among line members are not introduced until Sect. 4.5.

## 4.2 By-Pass Links for Error-Free Members

As has just been explained, decentralization in an organization is viewed here as a certain expansion of the graph that defines it structure. In a centralized organization, one node of the graph is distinguished from all others. It is a cut point whose removal disconnects all sources from all destinations. This node represents the Executive (or Exec, for short) of the organization. In a decentralized organization, there is no such cut point.

There are evidently many ways in which decentralization in this sense can be achieved, but they all entail the addition to the graph of branches which by-pass the Executive. This process can be carried to the point at which the position of the Executive loses most of its distinguishing characteristics, and perhaps is even abolished altogether. Nevertheless, it is convenient to begin here with a discussion of a decentralized organization in which the executive position is only slightly weakened.

In fact the first question one can ask is this. Under what circumstances is it advantageous to decentralize at all or, in other words, under what circumstances is the performance of an organization improved by introducing a link that by-passes the Executive? The propositions below, and their corollaries bear on this question.

The first deals with the addition of a by-pass link that goes from a staff member directly to a line member, such as the link labelled $u'$ in Fig. 4.1. Two structural arrangements are considered. One is centralized. Under it, the line member receives the transmission $u$ from the Exec, but none directly from the staff. The symbols of $u$ are related to those of the organizational input $x$ by conditional probabilities $p(u_j \mid x_i)$. They represent the equivocation SOP used by the staff vis-a-vis the Exec, as well as the assignment SOP used by the Exec for the line member, in this case Charlie. The fact that no transmissions go from the staff directly to Charlie under this arrangement is most conveniently expressed mathematically by a

fictitious symbol $u'_o$ which is sent to him but which means "no transmission".

The second arrangement is "slightly" decentralized. It allows for a symbol $u'_1$ to be transmitted to Charlie by the staff when $x_i$ was received from the outside and when the Executive sent to him $u_j$. However, $u_i$ can be transmitted only with the probability $\epsilon$ . Thus, under the centralized setup, the "by-pass SOP" for the staff is defined by the conditional probabilities

$$(4.1a) \qquad p(u'_0 \mid u_j, x_i) = 1, \quad p(u'_1 \mid u_j, x_i) = 0,$$

and under the decentralized one by

$$(4.1b) \qquad p(u'_0 \mid u_j, x_i) = 1 - \quad , \quad p(u'_1 \mid u_j, x_i) = \epsilon .$$

The next specification to be made is that of Charlie's response. Suppose that he is required to follow a pure SOP. For convenience, suppose that his putputs are labelled $y_{j0}$ or $y_{j1}$ where he received $u_j$ from the Executive, and $u'_0$ or $u'_1$ resp., from the staff. Suppose further that his output goes to the outside and that the organization reward is $L(i;j,0)$ in the first case and $L(i;j,1)$ in the second. (If $y_{jk}$, $k=0,1$, goes to an internal destination, $L(i;j,k)$ must be interpreted as an expected organizational reward which is conditioned on $x_i$ being the input and $y_{jk}$ Charlie's response.)

Under these circumstances, one can say the following:

<u>Proposition 4.1.</u> Suppose that a centralized organization transmits to a line member the symbol $u_j$ by way of the Executive with probability $p(u_j \;\, x_i)$ when $x_i$ is received, but by-passing the Executive only the "no-transmission" symbol $u'_0$. Suppose that this organization is decentralized by changing the by-pass SOP from (4.1a) to (4.1b). Suppose finally that the organizational reward

$$(4.2) \qquad L(i;j,1) > L(i;j,0)$$

for at least one $(x_i, u_j)$ - pair. Then, if the line and staff member are error-free, load-independent, and under-loaded in the centralized set-up, decentralization improves performance for sufficiently small $\epsilon$.

Proof. In the expression for the mean reward for the organization, only the two terms

$$L(i;j,0)p(u_0' \mid u_j, x_i)p(u_j \mid x_i)p_i + L(i;j,1)p(u_1' \mid u_j, x_i)\, p(u_j \mid x_i)p_i$$

are affected by the decentralization (4.1). Under the centralized set-up they are

(4.3a) $$L(i;j,0)\, p(u_j \mid x_i)p_i \;+ 0$$

and under the decentralized one

(4.3b) $$\begin{aligned} &L(i;j,0)\, p(u_j \mid x_i)\, p_i(1-\epsilon) + L(i;j,1)\, p(u_j \mid x_i)p_i\epsilon > \\ &> L(i;j,0)\, p(u_j \mid x_i)\, p_i(1-\epsilon) + L(i;j,0)\, p(u_j \mid x_i)p_i\epsilon = \\ &= L(i;j,0)\, p(u_j \mid x_i)p_i\, . \end{aligned}$$

Hence, an improvement is induced in the performance by decentralization unless overload is generated by it within the organization. However, this is not the case with the Exec since his SOP has not been changed. In Charlie's mean processing time

$$\tau_C = \Sigma_{r,s}\, t_{r0}p(u_0' \mid u_r, x_s)\, p(u_r \mid x_s)p_s + \Sigma_{r,s}\, t_{r1}\, p(u_1' \mid u_r, x_s)\, p(u_r \mid x_s)p_s$$

again only two terms are affected, namely the one in each sum which corresponds to the $(x_i, u_j)$-pair to which (4.1b) applies. These two terms are

$$t_{j0}\, p(u_j \mid x_i)p_i \;+\; 0$$

for the centralized arrangement, and

$$t_{j0}\, p(u_j \mid x_i)p_i\, (1-\epsilon) \;+\; t_{j1}\, p(u_j \mid x_i)p_i\epsilon$$

for the decentralized one. The second quantity can be made to differ as little as desired from the first, by choosing $\epsilon$ small enough. Hence, if Charlie is underloaded under the centralized organization, as was assumed here, he can avoid overload also in the dencentralized one. A similar argument applies to the staff member who processes u' to Charlie. The proposition is therefore proven.

The variational argument used in this proof (sometimes called a "Koopmans variation") is evidently good also when it is applied to an organization that is already decentralized. One would then assume that a by-pass SOP defined by a pair of probabilities $p(u'_0 \mid u_j, x_i)$ and $p(u'_k \mid u_j, x_i)$ for the transmission from the staff directly to Charlie is changed to

(4.4) $$p'(u'_0 \mid u_j, x_i) = p(u'_0 \mid u_j, x_i) - \epsilon\ ,\quad p'(u'_k \mid u_j, x_i) = p(u'_k \mid u_j, x_i) + \epsilon\ .$$

One can then readily prove

Corollary 4.1. Under the assumptions of Proposition 4.1, with (4.2) replaced by

$$L(i;j,k) > L(k;j,0),$$

the change (4.4) in the by-pass SOP improves organizational performance provided only $\epsilon$ is made small enough.

Proposition 4.1 and its corollary were derived under the assumption, among others, that the staff and line members who were involved in the decentralization process, were not load-dependent. Decentralization as they show, is then typically to the good unless those members are fully loaded to begin with. However, if they are load-dependent, this result may no longer hold. It is, on the contrary, possible that every change of the form (4.1) causes overload, and hence a potential performance deterioration. The reason for this lies in the fact that the mere addition of the symbol $u'_1$ to the input alphabet of a line member, and to the output alphabet of a staff member may cause overload, no matter how infrequently that symbol is transmitted.

To be more specific, consider once more the situation to which Proposition 4.1 applies. Denote Charlie's processing times under the centralized arrangement with $t_{r0}$, and with $t'_{r0}$ and $t'_{r1}$ under the decentralized one. Thus, $t_{r0}$ is the time Charlie requires for the production $y_{r0}$ when he need not expect any by-pass inputs but $t'_{r0}$ the corresponding time if he must be prepared for an occasional reception of $u'_1$. The next corollary gives a condition under which decentralization, no matter how slight, leads to overload on the line and hence to the risk of performance deterioration.

Corollary 4.2. Consider decentralization under the same conditions as those in Proposition 4.1, but assume that the affected line member is load-dependent. Let $\tau_C$ be his mean processing time under the centralized arrangement. Assume further that his processing times under the decentralization (4.1) obey

$$(4.5) \qquad t'_{r0} - t_{r0} > 1 - \tau_C , \qquad t'_{j1} - t'_{j0} \geq 0,$$

as $\epsilon \to 0$. The line member is then overloaded by the by-pass SOP (4.1b), no matter how small $\epsilon$. If more generally

$$(4.6a) \qquad t'_{rk} - t_{rk} \geq 0$$

holds, rather than (4.4b), and if it does so for all pairs (r, k) for which

$$(4.6b) \qquad L(i;r,k) > L(i;r,0) ,$$

all by-pass SOP's overload the line member.

Proof. Let $\tau'_C$ be the mean processing time of the line member Charlie under the decentralization set-up. Then, before the decentralization (4.1),

$$\tau_C = \Sigma_{rs}\ t_{r0}\ p(u_r | x_s)p_s + t_{j0}\ p(u_j | x_i)p_i ,$$

where the sum ranges over all (r, s)-pairs except r=j, s=i, and afterwards

$$\tau'_C = \Sigma_{rs}\, t'_{r0}\, p(u_r \mid x_s)p_s + t'_{j0}\, p(u_j \mid x_i)p_i(1 - \epsilon) + t'_{j1}\, p(u_j \mid x_i)p_i\,\epsilon\ .$$

Thus

$$\tau'_C - \tau_C = \Sigma_{rs}\ (t'_{r0} - t_{r0})\ p(u_r \mid x_s)p_s + (t'_{j0} - t_{j0})p(u_j \mid x_i)p_i$$

$$+ (t'_{j1} - t'_{j0})\ p(u_j \mid x_i)p_i\,\epsilon$$

$$> (1 - \tau_C)\ [\ \Sigma_{rs}\ p(u_r \mid x_s)p_s + p(u_j \mid x_i)p_i] = 1 - \tau_C$$

which shows that $\tau'_C > 1$, and that Charlie is overloaded for all $\epsilon$ as was asserted. This argument is evidently valid also for by-pass SOP's of the form (4.1) involving any two symbols $u'_0$ and $u'_k$. In that case, however, they apply also to a mixture of three, $u'_0$, $u'_1$, and $u'_2$ for instance. In fact merely one more term is introduced in the expression above for $(\tau'_C - \tau_C)$, namely one proportional to $(t'_{j2} - t'_{j0})\,\epsilon_2$ which is also non-negative. The same conclusion is thus valid also in this case and hence for mixtures of more than three by-pass symbols as well. This completes the proof of the corollary.

The substance of these results is that an organization of error-free members can fully expect to gain in performance from the addition of well-designed by-pass links from staff to line, provided only that no overload is introduced by them. In fact, it will gain if the information carried over those links is valuable to some line member, in the sense that it helps him occasionally in securing a higher reward for the organization than he could on the basis of the Executive's directives alone.

Such results seem a bit too good to be true. Organizations in practice are quite cautious in their use of by-pass links. If there is a rationale for caution, it must lie in the fact that the chief assumption made above is not really valid in practice. This is that the members are error-free. The next section bears this out.

## 4.3 By-Pass Links for Error-Prone Members

Overload, as the last corollary showed, may be an inevitable consequence of decentralization and so is therefore the incidence of error or malpractice. A deterioration of performance however will develop only in those cases in which the beneficial effect of decentralization is more than offset by the adverse effects of those errors. The trade-off is pertinent also when an organization member is error-prone even in the absence of overload. It is convenient to distinguish two cases; a simpler one in which it is assumed that the line member is error-free under the centralized arrangement but error-prone otherwise, and a more general case in which he is assumed error-prone under either arrangement.

In the first case one can derive a result which is a straight-forward generalization of Proposition 4.1, namely the following

Corollary 4.3. Consider decentralization under the same conditions as those in Proposition 4.1 but assume that the affected line member turns error-prone in the decentralized arrangement. The by-pass SOP (4.1b) then leads to a performance improvement if the conditional expectation

$$(4.7) \qquad E\ \{L \mid u_j,\ x_i\} \ > \ L(i;j,0)$$

Proof. Let the line member under consideration be again Charlie. Since he is error-free under the centralized set-up, he will respond with $y_{j0}$ when he receives $u_j$ from the Executive and $u'_0$ ("no transmission") from the staff. The expected reward to the organization will then contain the term (4.3a). However, since Charlie is error-prone in the decentralized arrangement, he will not necessarily respond with $y_{j0}$ or $y_{j1}$, depending only on whether he receives the message $u'_0$ or $u'_1$ from the staff. Instead, he will respond with some $y_{rs}$ from within the range of his possible outputs, and will do so with error probabilities $p'(y_{rs} \mid u_j, u'_0)$ and $p'(y_{rs} \mid u'_j, u'_1)$. The mean reward to the organization will thus contain not only the two terms on the left of (4.3b), but,

$$[\,\Sigma_{rs}\, L(i;r,s)\, p'(y_{rs} \mid u_j, u_0')\,(1-\epsilon) \;+\; \Sigma_{rs}\, L(i;r,s)\, p'(y_{rs} \mid u_j, u_1')\,\epsilon\,]\, p(u_j / x_i) p_i$$

$$= E\,\{\, L \mid u_j, x_i \,\}\quad p(u_j \mid x_i)\, p_i\,.$$

Thus, decentralization by way of the by-pass SOP (4.1) will improve performance only if this quantity is greater than the expression in (4.3a). This, however, is equivalent to (4.7).

The result (4.7) is quite plausible intuitively, especially if expanded into the form

$$[L(i;j,1)\, p'(y_{j1} \mid u_j, u_1') - L(i;j,0)\, p'(y_{j0} \mid u_j, u_0')]\,\epsilon \;+\; L(i;j,0)\, p'(y_{j0} / u_j, u_0')$$

$$+\; \Sigma'_{rs}\; L(i;r,s)\, p'(y_{rs} \mid u_j, u_1')\,\epsilon \;+\; \Sigma''_{rs}\; L(i;r,s)\, p'(y_{rs} \mid u_j, u_0')(1-\epsilon) >$$

$$> L(i;j,0)$$

in which the sum $\Sigma'$ omits the term for $r = j$, $s = 1$ and $\Sigma''$ the one for $r = j$, $s = 0$. The first term on the left then represents the gain in the organization performance due to Charlie's added information. The last two can be considered the dissipation of this gain due to Charlie's errors. This dissipation is the more serious the smaller the rewards $L(i;r,s)$, and the larger the error probabilities $p(y_{rs} \mid u_j, u_k')$ are that enter into those sums. The departure from the gain one might hope for, one the basis of Proposition 4.1, will be greater, the greater in magnitude the dissipation. In practice, most of those rewards $L(i;r,s)$ will in fact be negative, i.e. losses. Their magnitude is a measure of the risk implied by even slight decentralizations.

The result, on the other hand, is rather special. In effect, it assumes that Charlie's tendency to make errors is induced by decentralization, for instance, by overloading him. The more general case is that he is error-prone also under the centralized arrangement or, for that matter, that he follows a mixed SOP under it, rather than the optimal pure one, for some reason, good or bad. It would be desirable to have a condition which would indicate whether a change in a by-pass SOP will improve the organizational

performance or, conversely, whether such a change entails the danger of degradation. Such a condition can in fact be given, if one can make the (rather weak) assumptions of Sect. 2 regarding the load-dependence of organization members.

Proposition 4.2. Suppose that a change of the form (4.1) in the by-pass SOP to a line member changes his error probabilities from $p(y_{rs} \mid u_j, u_0')$ to $p'(y_{rs} \mid u_j, u_k')$, $k = 0, 1$. Assume further that the load-dependence assumptions of Sect. 3 hold for this member. Then, if the expected rewards, conditioned on $u_j$ and $u_k'$, increase more rapidly than the entropies, conditioned in the same way, performance will improve under such an SOP change. The same is true more generally for SOP changes of the form (4.4).

Proof. Let the line member be Charlie. Consider first the change in the conditional entropy from $H(y \mid u_j, x_i)$ to $H'(y \mid u_j, x_i)$, due to a change of the form (4.1) in the by-pass SOP. One has,

$$H(y \mid u_j, x_i) = -\Sigma_{rs}\, p(y_{rs} \mid u_j, x_i) \log p\,(y_{rs} \mid u_j, x_i)$$

$$H'(y \mid u_j, x_i) = -\Sigma_{rs}\, p'(y_{rs} \mid u_j, x_i) \log p'(y_{rs} \mid u_j, x_i)$$

where

$$p(y_{rs} \mid u_j, x_i) = p(y_{rs} \mid u_j, u_0')$$

because $u_j$ and $u_0'$ is the only information available to Charlie to begin with, and

$$p'(y_{rs} \mid u_j, x_i) = p'(y_{rs} \mid u_j, u_0')(1-\epsilon) + p'(y_{rs} \mid u_j, u_1')\,\epsilon\ .$$

This expression, together with the well-known concavity property of the entropy, gives

$$H'(y \mid u_j, x_i) \geq (1-\epsilon)\, H'(y \mid u_j, u_0') + \epsilon\, H'(y \mid u_j, u_1').$$

The load dependence assumption (3.2) now implies for the present case

$$H'(y \mid u_j, u_0') \geq H(y \mid u_j, u_0'),$$

$$H'(y \mid u_j, u_1') \geq H(y \mid u_j, u_1').$$

Hence,

(4.8) $$H'(y \mid u_j, x_i) \geq H(y \mid u_j, x_i).$$

Since Charlie is supposed error-prone under the initial by-pass SOP, the entropy $H(y \mid u_j, x_i) > 0$. It is then no restriction to assume further that

$$H(y \mid u_j, x_i) = E\{L \mid u_j, x_i\}$$

because H is defined only up to a constant factor which can be so chosen that H is equal to the conditional reward $E\{L \mid u_j, x_i\}$ under the same SOP. The corresponding reward under the changed SOP is

$$\begin{aligned} E'\{L \mid u_j, x_i\} &= (1-\epsilon) E'\{L \mid u_j, u'_0\} + \epsilon E'\{L \mid u_j, u'_1\} \\ &> (1-\epsilon) H'(y \mid u_j, u'_0) + \epsilon H'(y \mid u_j, u'_1) \\ &= H'(y \mid u_j, x_i) \\ &\geq H(y \mid u_j, x_i) = E\{L \mid u_j, x_i\}. \end{aligned}$$

The first inequality above represents the assumption of this proposition regarding the rapidity of increase of $E\{L \mid ., .\}$ compared with $H(y \mid ., .)$. The equation that follows utilizes a well known property of entropy, and the last inequality is (4.3). The first part of the proposition is accordingly proven.

The proof is clearly equally good if a symbol $u'_k$ other than $u'_1$ is added to Charlie's inputs, and regardless of whether or not it is the first or whether others were added before it. Therefore it remains to consider SOP changes of the form (4.4). These, however, do not entail an increase in the size of Charlie's input alphabet. According to the assumptions made in Sect. 3, there will be no change in the entropy of Charlie's error probabilities. The assertion of the proposition is therefore valid as a matter of course in this case. The proof is therefore concluded.

In practice, as mentioned at the end of the preceding section, considerable caution is exercized in how far decentralization is carried. The reason

for this may lie in the fact that the assumptions of this proposition fail beyond some point in almost all real situations. In other words, the expected rewards presumably often do not increase faster than the entropies of a member's error probabilities and malpractices. In fact, one can suspect that the former decrease while the latter continue to increase. The process of decentralization therefore should be expected to reach a point beyond which it cannot be carried without the risk of being counter-productive. The above proposition may contribute a rule by which this point might be roughly calculated in practice.

Propositions 4.1 and 4.2 as well as their corollaries are variational assertions. They apply under certain conditions under which a small change in the by-pass SOP will lead to performance improvements. On the other hand, they do not indicate which by-pass SOP is optimal. In this latter respect, the following holds.

Proposition 4.3. The best by-pass SOP for a line member is pure and given by the Bayes decision rule which maximizes $E\{L \mid u_j, x_i\}$, provided only this procedure does not overload him. If it does, the best SOP is mixed, in the way described in Propositions 6.1 and 6.2, [10].

A display of the proof is probably unnecessary here. The first part of the proposition is well known (see, e.g. [34], p. 124), and the second part is proven in [10], l.c. If $u_j$ is derived only from $x_i$ by some equivocation SOP, and not from any other source as well, it can of course be omitted from the conditioning of $E\{L \mid u_j, x_i\}$. On the other hand, if the Executive insists on transmitting $u_j$ to Charlie it will often be sufficient for the staff to supplement it with an equivocated version $u'_k$ of $x_i$; $u'_k$ would then be so chosen that Charlie could execute the SOP as if he received $x_i$ itself. It can then happen that the best equivocation SOP for the staff provides for no transmissions to Charlie at all or, equivalently for the transmission with probability 1 of $u'_0$. The symbols $u_j$ from the Executive are then in the nature of sufficient statistics for staff. As is well known (e.g., [28], p. 130),

there exist two functions $f_1$ and $f_2$ in that case such that

$$p(u_j, u'_k \mid x_i) = f_1(i, j)\, f_2(j, k) \tag{4.9}$$

This observation can also be stated as follows.

Corollary 4.3. The best organization structure for a line member is centralized if, under the SOP characterized by Proposition 4.3, the conditional probability of the transmissions to him of $u_j$ from the Executive and of $u'_k$ from the staff, for given organization input $x_i$, obeys (4.9).

Proposition 4.3, as well as its Corollary, hold regardless of whether the Executive is or is not error-free, provided only his error probabilities are known to the Designer. Under the same proviso, they hold also if the staff is error-prone. It is then merely necessary to replace $E\{L \mid u_j, x_i\}$ in the statement of the proposition with $E\{L \mid u_j, u'_k\}$. On the other hand, there is no allowance made in the derivation of the SOP for errors made by the line member. The SOP which is charactertized by Proposition 4.3, in other words, is the one which he should follow, and which he also could follow if he were not error-prone.

## 4.4 By-Pass Links from Private Sources

The discussion so far of decentralization in an organization has dealt only with by-pass links from the staff directly to the line. The implicit assumption, in other words, has been that all inputs to the line were intra-organizational. In practice, however, the line members of most organizations utilize inputs from sources of their own, which were called "private sources" in Sect. 4.1. These sources may be of various kinds, as was pointed out there. Some may supply intelligence of some kind but make it accessible to only one line member, or they may make available resources to him but to no one else. More frequently, however, they will represent the results of personal observation, or assessments of his immediate environment, or his personal experience and judgment regarding how he should best

do his job. These sources are represented by a block in Fig. 4.1, and the input from them to the line denoted with x''.

One might suppose that such sources should be treated quite differently from the intra-organizational ones. In particular, since they are private, one might argue that by their definition they should be exempt from interference by the organization. In particular, one may wish to exclude the possibility of some kind of equivocation by a staff member who would intercept x'' and replace it with a modified input u''. This is not so, however. Interference of this kind is not only possible but actually quite common in practice. A supervisor for instance, often places some strictures on his subordinates which amount to "Do as I say; don't do as you think", or else to some weakened version of the same thing. This is in effect an equivocation injected into the link from the private source, and the supervisor then acts in the role of an equivocating staff.

In the mathematical treatment of private sources, it is convenient to model the possibility of such equivocation by inserting a hypothetical organization member into that link, in the way shown in Fig. 4.2 and called a "Censor" there. The arrangement shown differs from the one of Fig. 4.1 in

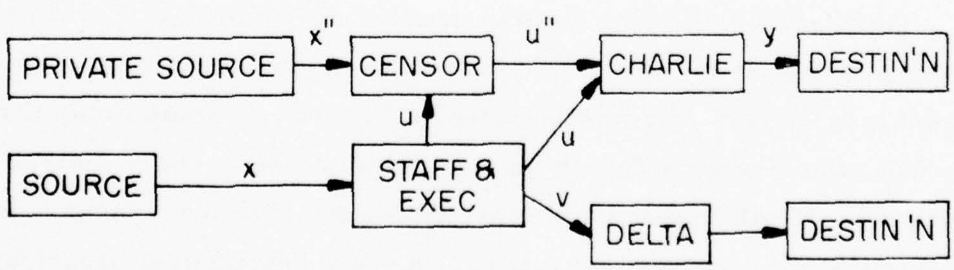


Fig. 4.2

Decentralization by Way of a Private Source

essentially only one way, namely, by the inclusion of the Censor. He can be considered another staff member or, as will become apparent in a later report as a member of the control branch. The staff and the Executive are combined in the figure and are shown as the transmitters of a single input u to the line member Charlie, rather than of the pair (u, u') of Fig. 4.1. This is evidently no restriction.

Under these conventions, however, it is clear that all results derived in the preceding two sections remain valid also in the case considered here, i.e. for by-pass links from private sources. In fact, if the Censor is interpreted as a member of the staff, the input u'' to Charlie plays the same role here as u' did in the preceding two sections. All results derived there, accordingly, carry over to the arrangement considered here. (This is still excluding cross links among the line. If they are admitted, a clear difference emerges between the treatment of intra-organizational and private inputs, as will be explained in the next section.)

One may wish to disagree with this conclusion. The argument in such a case might go as follows. The staff, in the present theory, is assumed to be capable of executing any kind of pure or mixed equivocation SOP. This assumption may be appropriate for the staff proper but hardly for the Censor. He may perhaps be considered, capable of acting like an on-off switch, either allowing the line member to use a private source or prohibiting him from doing so, but of no more subtle an SOP than that. This argument may be partly or wholly correct. It it is, it would imply that the theory will often assign SOP's to the Censor which he is not capable of carrying out. It will develop later that this is not a phenomenon that is restricted to Censors or, for that matter, to other hypothetical organization members of his ilk. Actual ones also typically fail to execute the SOP that is desired of them. They may do so either because they are incapable of carrying it out, or because they do not wish to do so, or because they have been given the wrong SOP by the Designer to start with. Whichever the

reason, any such failure is a malpractice, in the sense of the term defined in Sect. 3.2. The correction, insofar as it is possible or practical, is to be in another report on organizational control.

The conclusion is accordingly reached here that there is no essential difference in the theoretical treatment of intra-organizational and private inputs to the line, at least as long as cross links among the line members are not admitted. There is however a special case that is of interest here which was not pertinent earlier, namely the following. It can happen that the input u which, as Fig. 4.2 indicates, is normally sent to Charlie as well as his Censor, is useless to both. The Censor can then generate u'' from x'' without reference to u, and Charlie can generate y from u'' in the same way. In such a case, the organization decomposes into two independent units, each conducting its portion of the task without the need of coordination with the other. In one portion, Charlie is advanced to the position of an executive, with the Censor as his staff. In the terminology of graph theory, the graph of the organization is "disconnected". In the terminology to be used here, the organization undergoes a "dichotomy".

A sufficient condition for such an event is the following.

<u>Proposition 4.4.</u> Consider an organization receiving the symbols $x_i$ from its source, and with a line member whose private source symbols $x''_\ell$ are modified by a Censor to $u''_k$. Let this line member respond with $y''_{rs}$ when he receives $u_j$ from the Executive and $u''_k$ from the Censor, while the rest of the line responds with $y_t$. Assume that the organization reward $L(i, \ell; r, s, t)$ is additive, in the sense that

$$L(i, \ell; r, s, t) = L_1(i;t) + L_2(\ell; r, s).$$

The optimal organization then undergoes a dichotomy which separates the line member and his Censor from the remainder of the organization. If these two members are not load-dependent and if their processing times do not all exceed unity, their optimal SOP's are of the kind defined in Proposition 6.1 of [10]. That is, they are pure if they do not overload the line member and his Censor. On the other hand, if only mixed SOP's avoid overload, optimal ones require no more than two of the equivocation probabilities $p(u''_k \mid x''_\ell)$ and $p(y''_{rs} \mid u''_k)$ to differ from zero.

Proof. The mean reward for the organization, under the assumption of this proposition, is

$$E\{L\} = \Sigma [L_1(i;t) + L_2(\ell;r,s)]\, p(y''_{rs} \mid u_j, u''_k)\, p(u''_k \mid u_j, x''_\ell)\, p(y_t \mid u_j)\, p(u_j \mid x_i) p(x_i, x''_\ell)$$

where the sum ranges over all indices $i, j, k, \ell, r, s, t$. The probabilities under sum represent (in that order) Charlie's SOP for producing his output $y''$, his Censor's for producing $u''$, Delta's for producing $y$, and that of the staff and the Executive for producing $u$. The last factor represents the joint probability with which the two sources, the organization's and Charlie's private one, produce their symbols. This expression can evidently be written as the sum of two terms,

$$E\{L\} = \Sigma\, L_1(i;t)\, p(y_t \mid u_j)\, p(u_j \mid x_i)\, p(x_i) + \Sigma\, L_\ell(\ell;r,s)\, p(y''_{rs} \mid u''_k)\, p(u''_\ell \mid x''_\ell)\, p(x''_\ell). \tag{4.10}$$

They can be maximized independently of each other; the first with respect to the variables $p(y_t \mid u_j)$ and $p(u_j \mid x_i)$, and the second with respect to $p(y''_{rs} \mid u''_k)$ and $p(u''_k \mid x''_\ell)$. The dichotomy of the optimal organization thus takes place, as asserted. Both maximizations are subject to the usual overload constraints. However, neither the objective function nor all of the constraints are linear in their variables. (Some are bilinear.) One can nevertheless proceed as follows. Consider the second term. It is continuous in the variables, hence, assumes the maximum over the compact domain over which they range. Let $p^*(y''_{rs} \mid u''_k)$

and $p^*(u_k'' \mid x_\ell'')$ be a maximizing set, and suppose it underloads both Charlie and his Censor. Suppose further, that contrary to the assertion of this proposition, one or the other or both cannot be chosen in such a way that they consist of zeros and ones only. Suppose in fact that this is true of the $p^*(u_k'' \mid x_\ell'')$. In that case, one could solve a programming problem for the determination of another set of values $p(u_k'' \mid x_\ell'')$ which maximizes as the objective function the second term on the right of (4.10), subject to $p(y_{rs}'' \mid u_k'') = p^*(y_{rs}'' \mid u_k'')$, and to the constraints that the mean processing times of Charlie and his Censor must not exceed unity. It would be a linear programming problem because its objective function and both constraints are now linear in the variables $p(u_k'' \mid x_\ell'')$. The maximum would thus be attained at a point at which these variables have values of zero and one only. Hence, the values of $p^*(u_k'' \mid x_\ell'')$ could not have been optimal to begin with, which is a contradiction and bears out the assertion of the proposition for this particular case. The others are proven in analogous fashion.

This result can also be generalized to load-dependent members, as in Proposition 6.2 of [10].

## 4.5 Cross Links Among the Line

The conclusion reached above, in Sect.'s 4.2 to 4.4, singled out a particular line member Charlie and ignored all others. The line, in other words, was treated like a single-member line. This is no real restriction as long as the coordination among the line is executed entirely by their immediate supervisor, and not through some additional arrangments among the line members. Under this proviso, one can in fact show readily that the variational proof of Proposition 4.1 and its two corollaries carries over to an m-member line, regardless of whether it does its processing alternately or in parallel, provided the operation is error-free. The results obtained earlier now hold member by member. The same applies to the generalizations in Corollary 4.3 and Proposition 4.2, in case one or more of the line members are error-prone, and to the remarks in Sect. 4 regarding the admission of inputs from private sources.

The conclusions in fact apply also under certain circumstances when cross links exist among line members and direct coordination between them is permitted. This is so when the operation of the Executive is error-free and when the line members are prohibited from using private sources. If, in other words, the line has access only to error-free intra-organizational sources, one can use the reasoning of Sect. 5.6 of [10] to show that cross links among the line members are superfluous: they cannot improve performance and they may in fact worsen it. The reason for this is, roughly speaking, that if all inputs to the line members are intra-organizational, an error-free staff and Executive will see to it that those inputs provide all that is needed for the proper execution of the line jobs. No line member can then supply to another any inputs that will not have already been supplied to him by the staff or the Executive. Cross links among the line are thus unhelpful under these conditions, and they may be harmful because they can cause overload where there was none without them.

It thus remains to discuss the conditions under which an improvement is possible, i.e., when a line unit allows its members to use their private sources, or when an Executive/staff combination is error-prone.

The first situation, i.e. the admission of private sources can be treated in ways similar to those followed in the preceding section. Consider, for simplicity, the line of Fig. 4.2 in which both members, Charlie and Delta, have access to private sources, if by way of Censors. However, let a cross link be added going from Charlie to Delta and carrying a signal $z$, as well as a second one going the other way and carrying $w$.

In order to simplify notation, denote with $u$ the combined inputs to Charlie, intra-organizational as well as private, and $v$ with those to Delta. Suppose both to be load-independent and error-free. Four "cross link" SOP's will be considered. Under the first, no signals are passed between them or, which is saying the same thing, Charlie is restricted to the transmission with probability 1 of a symbol $z_0$, meaning "no transmission", when

he receives $u_j$, and Delta is similarly restricted to $w_0$ when she receives $v_k$. Thus, under the SOP,

$$p(z_0 \mid u_j = 1), \quad p(w_0 \mid v_k) = 1. \tag{4.11}$$

Under the remaining three cross links SOP's, two additional symbols $z_1$ and $w_1$ are transmitted when $u_j$ and $v_k$ are received, either one way from Charlie to Delta or in the opposite directions, or both ways. These SOP's are more specifically defined by

$$\begin{aligned} p(z_1 \mid u_j) &= \epsilon_C, & p(w_1 \mid v_k) &= \epsilon_D, \\ p(z_0 \mid u_j) &= 1 - \epsilon_C, & p(w_0 \mid v_k) &= 1 - \epsilon_D \end{aligned} \tag{4.12}$$

in which $\epsilon_C = 0$ or $\epsilon_D = 0$ represent one-way, and $\epsilon_C \neq 0$, $\epsilon_D \neq 0$, two-way transmissions.

Suppose finally that Charlie, on receiving $u_j$ jointly from the Executive, the staff and his private source, plus $w_\ell$ ($\ell = 0, 1$), from Delta is required to produce an *output* $y_{j\ell}$. Similarly, Delta on receiving $w_m$ ($m = 0, 1$) from Charlie and $v_k$ from all other sources, responds with an output $y'_{km}$. Let the (conditional) reward under these input/output arrangements be $L(j, k; \ell, m)$ and suppose that there exists a pair $(u_j, v_k)$ of inputs to the line for which

$$L(j, k; \ell, m) > L(j, k; 0, 0) \tag{4.13}$$

for either $\ell = 0$, $m=1$ or $\ell = 1$, $m=0$. In that case, one can say the following.

<u>Proposition 4.5.</u> Consider an organization consisting of error-free members which permits its two-member line the use of their private sources. Assume that the line is load-independent and underloaded. Assume further that the conditional reward to the organization obeys (4.12) when the line members receive $z_\ell$ and $w_m$ from each other, and $u_j$, $v_k$ (with positive probability) from all other sources, respectively. The performance of this organization

is improved if the cross link SOP is changed from (4.11) to (4.12) with either $\epsilon_C = 0$ or $\epsilon_D = 0$. If (4.13) holds for $\ell = 0$, $m = 1$ as well as for $\ell = 1$, $m = 0$, and if also

$$L(j,k;1,1) > L(j,k;0,0), \tag{4.14}$$

an improvement is achieved if $\epsilon_C \neq 0$, $\epsilon_D \neq 0$. If neither of the two line members has access to private sources, no cross link SOP leads to an improvement.

Proof. The proof of this result is analogous to that of Proposition 4.1. One selects from the expression for the conditional reward the four terms that are affected by the change in the cross link SOP which are

$$\Sigma_{\ell m} \; L(j,k;\ell,m) \; p(z_\ell \mid u_j) \, p(w_m \mid v_k). \qquad (\ell, m = 0, 1).$$

Then, if for instance $L(j,k;1,0)$ satisfies (4.13), the cross link SOP (4.12) with $\epsilon_D = 0$ gives

$$L(j,k;0,0)(1-\epsilon_C) + L(j,k;1,0)\,\epsilon_C > L(j,k;0,0)$$

which is a performance improvement. The cases in which $L(j,k;0,1)$ satisfies (4.13), and the one in which (4.13) holds as well, are verified similarly The proof, finally, of the uselessness of cross links is the same as of Proposition 5.5 of [10].

The parallel between Proposition 4.1 and this one can be continued. One readily generalizes the last result to variational changes in the cross link SOP, analogous to those considered in Corollary 4.1. One can further show, as in Corollary 4.2, that load-dependence among the line may negate the benefit of cross links there. One can also establish counterparts to Corollary 4.3 and Proposition 4.2, and demonstrate that error incidence or malpractice among the line can lead to a similar negation. This parallel is evidently valid for lines with an arbitrary number of members, rather than merely the two mentioned in Proposition 4.5.

The parallel fails when it comes to the optimal SOP which was characterized for by-pass links in Proposition 4.3. The analogous characterization for cross link SOP's is not necessarily optimal. (In the terminology of team theory, see [21], p. 138), it is merely "person-by-person satisfactory".) The source of the complication is essentially the same as that encountered in Sect. 6 of [10] which dealt with the design of the organizational staff, namely the need for coordination. This need did not arise when the staff did its processing alternately (see Sect. 6.4, [10]), nor does it arise on the alternately processing line for essentially the same reason. Coordination on the other hand is needed among a parallel processing staff in order to insure the joint production of an optimal output (see Sect.'s 6.5 and 6.6, [10]). This implied the use of cross links for two purposes. One was the distribution among the staff of the inputs received by the members from their sources. The other was the allocation among the members of the various portions of the output production. The cross links over which the inputs are distributed were found to be basically symmetrical. For optimality, any staff member should in principle be able to transmit the inputs from his source to all other members, and only considerations of irrelevance or overload can negate that rule. The allocation of the output portions, on the other hand, were shown to be often unsymmetrical (see Sect. 6.5 of [10]). One member, the "Chief of Staff", chose the output symbol that is to be generated. The various portions of the job were then opted for sequentially, starting perhaps with him but in any case one by one by the remaining members.

The analogy is probably quite apparent between this kind of staff operation on one hand, and that of a parallel processing line with private sources on the other. The members of such a line receive their inputs from several sources. For optimality, they should be able to pool these inputs and, again, only irrelevance or overload considerations should negate this rule. Thus, cross links should be available over which the pooling can take place. In fact, there is no reason why the Executive should be excluded. There should, in

other words, exist links over which the line members can transmit to him whatever inputs each has received from his private sources which are relevant to their joint performance and which do not overload the Executive. These links will be called "check-back links", for a reason to be explained elsewhere.

Cross links may have to be provided also over which the allocation of the various portions of the output can take place. This allocation, just as its counterpart on the staff, requires one member to choose the output that is to be jointly produced. On the staff, this was done by its Chief. On the line, it is natural (but not inevitable) that the Executive make this choice. According to [10], he can now use either of two procedures for the distribution of the output portions among the line. One is an assignment SOP of the kind described in Sect. 5.4 of [10], and the other is the sequential allocation mentioned above. The latter places a smaller output load on the Executive but a larger one on the line members. The Designer will accordingly have to choose between the two options on that basis. Suppose that he chooses the former. (There is actually a third option which is in effect a cross between these two and which will be described below in Sect. 6.2. The same three procedures are open also for the Chief of Staff, a fact which is overlooked in [10].)

The graph of a two-member line operating in this fashion is shown in Fig. 4.3. Charlie, for instance, receives three inputs: u from the Executive, u'' from his private source, and w from Delta. The first is his job

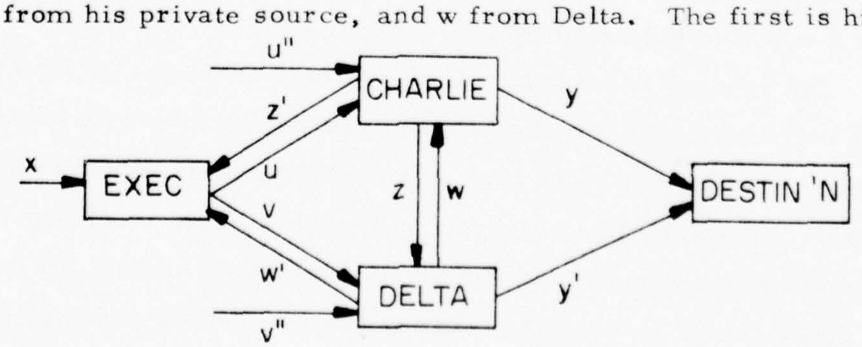


Fig. 4.3
Parallel Processing Line Group

assignment, the second e.g. his personal intelligence, and the third the relayed version of Delta's personal intelligence. He produces three outputs. One goes over his cross link transmission to Delta and the other over his check-back link to the Executive. Both convey what is relevant of u''. The third, namely y, is his organizational output which goes to a destination further down the line or outside.

The various interchanges among this triplet are very similar to those in a sociological unit of three or more members often referred to as a "group" or a "small group" [25]. This is suggested by the caption of the figure. A multi-echelon line made up of such groups has been advocated by Likert [19] as ideal for modern organizations, though not obviously for the reasons that led to it here, namely the desire for optimal performance.

Such performance is achieved by pure SOP's for the Executive and the line, provided they avoid overload everywhere, and by certain mixed SOP's otherwise. A precise statement regarding the nature of these SOP's is apparently too involved to be overly useful. A partial characterization is provided by the following proposition.

Proposition 4.6. Consider an operational mode of the kind illustrated in Fig. 4.3 in which the Executive receives the complete private inputs of all line members over check-back links and makes the task assignments accordingly. In this mode of operation, cross links among the line are superfluous unless the Executive is error-prone or overloaded, regardless of whether the line processes alternately or in parallel. The best SOP for the Executive is then a Bayes decision rule which selects the optimal organizational output on the basis of all inputs he receives, and which assigns the output production according to the rules developed in Sect. 5 of [10]. If the Executive is error-prone and the line operates in parallel, the best SOP for the line is the same as for a parallel processing staff which pools the Executive directives with the inputs from all private sources. It chooses the optimal output symbol by a Bayes rule and allocates its production as described in Sect. 6 of 5.

Proof. The first assertion, regarding the superfluity of cross links, is clearly correct if the Executive is neither overloaded by the messages from the line nor error-prone. He then in effect acts as a set of cross links. If he is overloaded by them, however, equivocation may be required to rectify the situation. The cross links among the line then distributes whatever is kept from the Executive. If he is error-prone, these cross links can further provide some redundancy in the messages that reach the line members. Their optimal SOP is then a Bayes decision rule, as is well known ( [34], p. 124). The same applies to the Executive, assuming he is not overloaded by that rule. Both are therefore as asserted in this proposition.

These observations will be further elaborated on in Sect. 6.2.

# 5. THE "$\psi$-DEPENDENT" ORGANIZATION MEMBER

## 5.1 $\psi$-Dependence

The performance of an organization member, i.e. the quantity and quality of his work, depends on factors other than those called "load factors" above. Load factors are intended to characterize the difficulty of his task. The others are to be called "$\psi$-factors" here. They characterize the motivation of a member towards the execution of his task, at least to the extent to which it can be influenced by the structure and the operating procedures of the organization.

The fact that the performance of a member depends, in the long run, on his motivation and the various ways in which it might be enhanced, has received considerable attention during the last several decades, and much study has been devoted to the question of what set of variables may be used to characterize this effect. Franklin, in a recent review of the literature [14], counts thirty-six candidates that have been identified by various authors and with various degrees of clarity. These factors are patently not independent. Some are almost certainly dependent on others, and changes in one evidently often induce changes in some others. This suggests that the list can be shortened, and perhaps even substantially so. A further shortening is possible here because some of the variables represent characteristics of individuals or groups which, by all indications, cannot be influenced by any features of organizational design, structural or procedural. Such characteristics are not germane to the subject of this report, and need not be considered.

Further omissions are possible for a different reason. These are all factors which are influenced primarily by dynamic (i.e. transient) phenomena in an organization, or else factors whose primary effect is on such phenomena

These are excluded from the present report, by a standing assumption (see Sect. 2).

One can hope that the list of factors may be fairly short, and indications are that it is. Porter, Hackman and Lawler [26] (p. 42) in fact reduce it to three items. The motivation of an organization member toward his task, and his performance in it, is a function of his feelings in three respects, according to these authors, namely these:

(a) the degree of autonomy that is accorded to him;

(b) the status which he feels he occupies vis-a-vis the organization and his peers; and,

(c) the extent to which he is satisfied by the nature of his job.

It will develop below that item (b) in this list is not a single factor, but actually a vector of at least three. This may account for the difference between this list and another which was established for roughly the same purpose, but, so it seems, independent of the one above. This is Vroom's expectancy theory (see, e.g. [17], for a recent review of it). It actually goes a step beyond a mere listing of the factors that affect motivation, and hence performance. It actually proposes a mathematical formula which expresses an organization member's motivation as a function of certain variables. Among these, one can probably identify one with item (c), namely job satisfaction, in the list above, and a second variable as part of item (b). Item (a) is missing from Vroom's formula and so is the remainder of (b). On the other hand, there is one variable in that formula which is missing in the above list. This is the expected utility to the organization member, of success with a task in which failure is possible. Observational evidence, however, does not clearly

support the inclusion of this factor. It will therefore be ignored below.

In other words, in what follows, the three factors listed by Porter, Hackman, and Lawler will be treated as the three most significant motivational $\psi$-factors which are not connected mainly with the dynamic phenomena in an organization. They will be re-formulated mathematically in what is hoped to be a plausible and uncontroversial way.

The purpose of the discussion which follows is the conversion of this list to a mathematical form. This is not a trival undertaking. Although the meaning of such terms such as "autonomy", "status", and "job satisfaction", may seem quite plain intuitively it is hardly obvious how to define them quantitatively. Definitions must in fact be made in such a way that they involve only variables and concepts that have already been introduced into the theory, and they must be arrived at in a reasonably convincing fashion.

This is attempted, under the heading of "autonomy" for item (a) on the list. It develops to be the most interesting of the three, from the standpoint of organizational design (and also from that of utility theory). It introduces design considerations which had quite obviously been missing from the theory so far, and which are equally obviously present in practice. Organization members develop to be "trust seekers" or "trust shunners" or somewhere between the two. Sect. 5.3 deals with "organizational status" and finds that it is best resolved into three factors which seem largely independent of each other. One of these comprises the incentives of an economic nature which are available to an organization for the motivation of its members. The other two are the proportions of time which the supervisor and the colleagues (peers) of a member may

allocate to him. Sect. 5.4, finally, treats the concept of "job satisfaction" in roughly the sense in which the present-day proponents of work re-design use it. The conclusion is reached that the concept fits neatly into the framework of this theory. It simply tends to favor alternate processing over parallel (using the terminology of Sect. 5 of [10]).

## 5.2 Autonomy

The objective of this section is to present a mathematical version of the qualitative assertion according to which "the performance of an organization member tends to improve as the degree of his autonomy increases".

The first thing to be made specific is what is to be meant by the "autonomy" of an organization member, such as Charlie, in Fig. 5.1. He is the member of a decentralized organization since he receives his inputs not only from the Executive but also directly from the staff, as well as from a private source by way of

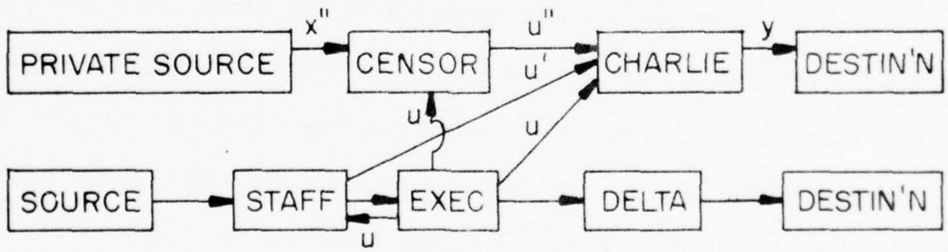


FIG. 5.1

AUTONOMY OF AN ORGANIZATION MEMBER

his "Censor" whose presence in the link to this source was explained in Section 4.4. Will Charlie feel that he has been given some autonomy, and under what circumstances will he do so? The answer presumably is, at least roughly speaking, if his intra-

organization inputs u and u' do not completely determine the action, y, which he should take, or, in other words, if he is left some latitude in the choice of the SOP which he follows in the selection of his response.

Charlie may be working under two possible organizational arrangements. Under one, he would be prohibited from using any inputs other than intra-organizational ones, namely u and u'. His censor would then simply block all access to his private source. He may still feel that he has some autonomy under this arrangement, but it is likely that he will not be overly elated by it. It prohibits him from utilizing his personal judgement, his experience, his familiarity with the immediate environment, etc., all of which are components of his private source, as was explained in Sect. 4.1. In effect, his autonomy under this arrangement consists of his being allowed to choose among the options that are open to him by tossing a coin after he has received his intra-organizational inputs. He has, after all, no other basis for his choice under the circumstances. The issue of autonomy therefore is unlikely to be very consequential unless access is permitted to his private sources.

Suppose accordingly that Charlie has such access. He will then presumably assess the degree of his autonomy by the range of the SOP's which he may use in the choice of his response to the inputs he receives, the intra-organizational ones from the Executive and the staff, as well as those from his private source. This statement is valid in essence also if the inputs from his private source are subject to modification by his Censor. The range of his SOP's will often include mixed ones but will rarely include all possible mixtures. For example, he will no doubt be restricted to SOP's which overload neither him nor his subordinates.

The range of his SOP's, however, may not be the only consideration in Charlie's assessment of his autonomy. He may, for instance, find that the organ-

ization has given him latitude only among options which are of little or no consequence to its performance. Charlie may then feel that his autonomy is trivial and derive no satisfaction from it. His individual performance will then hardly be enhanced by it. He may, however, react also in the opposite way. He may be delighted with the fact that he need not worry over the consequences to the overall organizational performance of what SOP he decides to adopt. In other words, Charlie may attach personal utilities to the options that are open to him and rate his autonomy by the range of the utilities which the various admissible SOP's allow him to expect.

One can put this into mathematical terms in the following way. Suppose that Charlie receives the input symbols $u_i$, $u_j'$, $u_k''$, from his Executive, from the staff, and from his Censor, respectively, and that he responds with $v_\ell$. He will then attach some value $L_C(i,j,k;\ell)$ to this input/output combination. On the basis of what has just been said, he will then assess the "degree of his autonomy" by the quantity

$$(5.1) \qquad a = \max_p E\{L\} - \min_p E\{L\}$$

where p has been written for the probability $p(v_\ell \mid u_i, u_j' \, u_k'')$ which defines one of the SOP's Charlie is permitted to adopt for himself.

The value $L_C\,(i,j,k;\ell)$ which Charlie attaches to a particular input/output combination may coincide with the organizational reward $L(i,j,k;\ell)$ that is expected for his position. He is then a member who is best pleased with a wide range of potential consequences that may result from his actions. He enjoys the trust that is implicit in such an arrangement and reacts correspondingly. He is

what might be called a "trust seeker". His personal performance, his processing times $t_{ik}$ and the entropy H of his errors or malpractices are then presumably monotone functions of $\alpha_C$, calculated with L in place of $L_C$. A "trust shunner", by contrast, might attach the utility $-L_C(i,j,k;\ell)$ to the same input/output combination and calculate the degree $\alpha_C$ of his autonomy with $L_C = -L$. Many other utility functions $L_C$ can be equally well visualized and equally well accomodated. Utility theory [13] guarantees that, under the assumption made in this study, such a function exists provided only that Charlie's preferences define a "weak order" on all possible input/output combinations.

Regardless of which $L_C$ is used in (5.1), the following statement can be made regarding the degree of autonomy $\alpha_C$.

Proposition 5.1. The degree of autonomy, as defined by (5.1), is a convex function of the equivocation probabilities $p(u_i, u_j' \mid x_r)$ used by the Executive and the staff when those of the $p(u_k'' \mid u_i, u_j', x_s'')$ used by the Censor are fixed, and vice versa.

Proof. For convenience of notation, assume that there are no direct inputs to the organization member Charlie from the staff. The equivocation probabilities are then more simply $p(u_i \mid x_r)$ and $p(u_k'' \mid u_i, x_s'')$. Let the latter be fixed. Write $E'\{L_C\}$, $E''\{L_C\}$, and $E\{L_C\}$ for the expectation of $L_C$ calculated when the Executive uses $p'(u_i \mid x_r)$, $p''(u_i \mid x_r)$, and

$$p(u_i \mid x_r) = \lambda p'(u_i \mid x_r) + (1-\lambda)p''(u_i \mid x_r)$$

respectively. The corresponding degrees of autonomy are $\alpha'_C$, $\alpha''_C$, and $\alpha_C$. Then, e.g.

$$\max_p E\{L_C\} = \max_p[\lambda E'\{L_C\} + (1-\lambda)E''\{L_C\}]$$

$$\geq \lambda \max_p E'\{L_C\} + (1-\lambda)E''\{L_C\}$$

where the equality above holds because of the linearity of $E\{L_C\}$ in the $p(u_i \mid x_r)$, and the inequality because the maximum taken over the two terms separately can be no less than that taken over their sum. Similarly,

$$\min_p E\{L_C\} \leq \lambda \min_p E'\{L_C\} + (1-\lambda)E''\{L_C\}.$$

It then follows immediately that

$$\alpha_C \geq \lambda\alpha'_C + (1-\lambda)\,\alpha''_C\ .$$

Since the reasoning used here clearly applies also to the equivocation probabilities $p(u''_k \mid u_i, x''_s)$, the proposition is proven.

Its usefulness lies in the fact that this geometrical property of $\alpha_C$ often carries over to the performance of the individual organization member. The fact is more particularly the following.

<u>Proposition 5.2.</u> Assume that the processing times $t_{ik}$ of a load-independent organization member, and/or the entropy of his error or malpractice probabilities, are not only monotone non-increasing but also concave functions of the degree of autonomy. They are then concave also in either set of equivocation probabilities $p(u_i \mid x_r)$ and $p(u''_k \mid u_i, x''_s)$ of the Executive and the Censor, when the other is fixed.

<u>Proof.</u> Let Charlie be again the organization member. Since he is load-independent his processing times $t_{ik}$ are function of $\alpha_C$ only. By a well-known

theorem (e.g., [27] (p. 20)), the convexity of $\mathcal{I}_C$ in the equivocation probabilities implies the concavity of the $t_{ik}$, under the assumptions of this proposition. The same is evidently true of the entropies $H(v \mid u_i, u''_k)$ of Charlie's error or malpractice probabilities, as was to be proven.

The result can be extended to a load-dependent or organization members if one can assume that his processing times $t_{ik}$ are concave jointly in the input probabilities $p(u_i, u''_k)$ and $\alpha_C$.

The definition (5.1) of the degree of autonomy seems fairly satisfactory conceptually. It would have been nice to derive it from a set of intuitively plausible postulates, in the way the notion of a utility function is established inutility theory [13], and an attempt was in fact made in that direction. It was not successful, however.

One can criticize the definition for an assumption which is, strictly speaking, inconcsistent with it. This is especially true for the type of organization member who was called a "trust seeker" above. The assumption is that one can calculate an expected reward $L(i,j,k;\ell)$ for every position in an organization, such as Charlie's in the discussion above. This reward, however, can only be calculated if the joint probability is known with which the symbols $u_i$, $u'_j$, and $u''_k$ arrive at his position, and, if he is not the producer of the final organization output, also the probability with which such an output is ultimately generated when he produces $y_\ell$. None of the probabilities, however, are precisely known if Charlie's predecessors and successors have autonomy. They have, after all, been given some latitude in how to choose them. Charlie may then feel that any latitude accorded to others is an infringement on his

autonomy. He would then add a "min" to his definition of $a_C$ in (5.1), and insist on taking the minimum over all SOP's among which the other organization members may choose. It has been suggested [23] that Charlie, if he takes this point of view, should be opposed to any autonomy granted to others because it detracts from his "power" in the organization. Such considerations introduce very interesting competitive aspects into organizational design. However, they were not considered in this study so far.

The observational evidence relating the degree of autonomy to the productivity of an organization member is very scant. In fact, only one study is known to this writer [31] and that is negative. It finds no appreciable increase in productivity of research workers due to an increase in their autonomy. This may be true of research workers but it is certainly contrary to experience in many other situations. A typical new supervisor, for instance, is well known to react to his freshly won autonomy by taking his work home with him. This voluntary overtime leads to an increase in his productivity which is much too large to be non-observable. There seems to be good reason in other words for its inclusion in this study.

5.3 Organizational Status

The second factor to be discussed here, as a contributor to an organization member's motivation and performance, is his assessment of the status which he occupies vis-a-vis the organization and his peers. This factor will actually matter here only to the extent to which the status can be influenced by the design of the organization, i.e. by its structure and by its practices.

A member will presumably feel that he is being rated highly within his organizational environment if he finds that certain scarce or valuable resources are being made available to him. In fact, he is likely to consider himself the better appreciated, the more of these resources are extended to him or the greater their value. An organization, however, has basically only two types of scarce resources. One is money, and the other is time. Money can be turned over to a member in the form of incentives, such as salary raises, bonuses, stock options, etc., or in certain equivalents, for example as special services, special accommodations, or special privileges which entail expenses to the organization. Time can be made available to a member either by his superiors, or by his peers, or by his subordinates, but only the first two are likely to contribute positively to what he considers his status.

The conclusion, therefore, seems to be that the organizational status of a member is determined mainly by three variables, namely by the value he attaches to the economic incentives bestowed on him,by the time his supervisor, and by the time his peers, allocate to him. The first of these perhaps should be resolved into several variables, each representing one of the many forms in which incentines may be cast. Vroom's expectancy theory [17] in particular distinguishes an arbitrary number (but does not identify them).

For the present purpose, this distinction appears to be unnecessary. In fact, the allocation of money or, for that matter, costs of any kind have been disregarded in this study so far, and their effect of the design of an organization has been ignored. In any case, a thoroughly satisfactory rationale for the allocation of economic incentives has been provided by J. Marshak [22]. Thus, only the allocations of time will be considered here because they have an effect on the design of an organization, as will be explained in Section 6.3.

The proportion $\beta_1$ of the time which a supervisor spends with his subordinates, individually and/or collectively, seems to have a positive effect on their productivity. That is, indications are that both the quality and the quantity of their work improve as $\beta_1$ increases. One can perhaps surmize further, that the general economic law of the diminishing marginal returns applies and that the rate of improvement diminishes as $\beta_1$ increases. One can no doubt further that the rate also depends on the nature of the work of the subordinates and on the way the supervisors use their time allocation. (The latter may, in fact, be equivalent to the rather well-worn "leadership" concept.)

The situation is roughly the same with respect to the proportion $\beta_2$ of time allocated to a member by each of his peers. In this regard, indications are that there is again a positive effect on productivity, perhaps not so much in the quantity as in the quality of his work. This may be especially true if absenteeism and turn-over rates are allowed for. The effect, however, appears to be substantially weaker in any case, and the law of marginal returns more pronounced. In fact, it is known that, in many cases, a reversal takes place and improvement turns into degradation because of collusion among the subordinates

towards lower performance norms [5] (p. 67).

A reversal is likely to take place also for a simpler reason, namely, the following. Suppose that a member, say Charlie, is fully loaded under an arrangement under which no time is allocated to him by his peers. I.e., $\beta_2 = 0$. Suppose next that a change is made and that he, as well as each of his peers, devote a proportion $0 < \beta_2 < 1$, of the unit time interval to their interaction with each other. Let Charlie's mean processing time be $r_C = r_C(\beta_2)$. Then ${}_C(0) = 1$, and if overload is to be avoided when $\beta_2 \quad 0$,

(5.2) $$r_C(\beta_2) \leq (1 - \beta_2) = (1 - \beta_2)\, r_C(0).$$

This means that, as $\beta_2 \rightarrow 1$, $r_C(\beta_2)$ would have to approach zero. Each member would have to be so utterly motivated by time spent with his peers that he would do his organizational job in virtually no time at all. This will rarely be possible. The relationship (5.2), in other words, will typically be violated above some critical value of $\beta_2$, and the trend towards performance improvement will be reversed. In fact, one can suspect that this critical value will be quite small in many situations in practice.

Although the literature on organizational psychology is pregnant with books and papers which bear out the beneficial effect of the two factors discussed here, relatively little seems to relate it in a quantitative way to the productivity of an individual, and most of that seems to be somewhat outdated. (See e.g., [32], for a review of the literature up to 1958.) This may not be a serious defect regarding $\beta_2$ whose influence may well be rather weak, as has just been pointed out. On the other hand, the $\beta_1$-factor seems important. The pro-

portion of the time which a supervisor spends with his subordinates seems to have a fairly strong influence on their morale. It may, in fact, be more important than the way in which he spends it (a question which will be discussed as part of the problem of organizational control).

Both proportions $\beta_1$ and $\beta_2$ are likely to vary also with the work that is being done in the group. Performance in routine work may not be greatly affected by either. On the other hand, the performance of a group of researchers may be quite sensitive to both.

The effect of the two factors on organizational design is discussed in Section 6.3.

## 5.4 Job Satisfaction

During the last two decades, approximately, there has been a growing awareness among the observers of life in organizations, of the deadly monotony and repetitiousness of many jobs. A movement accordingly got under way which was aimed at re-defining tasks in such a way that they would be more varied, more meaningful, and more satisfying to those who had to execute them. The trend seems to be strongest in the Scandinavian countries (see, e.g. [ 3 ]) where the goal is the achievement of "industrial democracy", a concept that apparently includes not only the factor to be discussed here but also those of the preceding sections. In the U.S., the objectives of the movement seem to be better separated. One speaks of "job enhancement", and "work design" here (see, e.g., [26] p.274). This section is concerned with the effect on the performance of an organization member that can be expected from this, more limited, kind of effort.

The basic ideas of it are that many workers perform better if they execute well-rounded tasks with a tangible end product, and with a well-identified re-

cipient for that product. This is a conscious reversal in the thinking of industrial engineers whose goal had been the resolution of every job into as many and as simple sub-tasks as possible in order to exploit the efficiency derived from the highest possible degree of specialization. The pursuit of this goal developed to achieve the exact opposite, namely, worker disaffection, absenteeism, and turn-over which more than cancelled the expected efficiencies. It was accordingly concluded that a return to larger and better rounded tasks would lead to an increase in productivity, at least in the long run.

A review of the literature, (e.g., [8]), which reports on the experience with the re-design of jobs, however, does not indicate that a clear improvement of performances is being obtained in terms of output quantity. On the contrary, the evidence points, if anything, to a slight decline. (This may be due to the fact that inadequate allowances are being made in these reports for reductions in absenteeism and turn-over.) On the other hand, there seems to be a fairly clear-cut case for an improvement in quality. The incidence of errors or malpractice is quite definitely reduced.

The idea of job re-design is, in any case, readily introduced into the theory. First of all, it probably applies mostly to the line and in fact mostly to the lowest line echelon, (to the "line operators", in the terminology of [10]) where the risk of work tedium is greatest. This work can then ususally be executed in two different ways, by "alternate" and by "parallel" processing (see [10], p. 42). Alternate processing typically implies larger task units, parallel processing small ones. The idea of work re-design then is, roughly speaking, that the processing times and the error entropies, for the same task and the same operators, are lower under alternate than under parallel processing.

To be more specific, suppose that a two-member line unit is called upon to process a symbol $u_k$ which is in fact a pair $(u_k^1 , u_k^2)$ of two sub-symbols. The processing can then be done alternately in which case $u_k$ is assigned to one operator, Charlie for instance, who requires the time $t_{Ck}$ for the job. It can also be done in parallel, however. If Charlie is then assigned the task of processing the sub-symbol $u_k^1$ , he has no assurance that he will be called upon to process also $u_k^2$ . That may be turned over to Delta (perhaps because she is expected to do it more efficiently). Under these conditions, Charlie will take the time $t_{Ck}^1$ to process $u_k^1$ and, if he gets the second subjob as well, $t_{Ck}^2$ to process $u_k^2$ . The idea of work re-design then in

$$t_{Ck} \leq t_{Ck}^1 + t_{Ck}^2$$

i.e., that Charlie will take longer to do a job piecemeal. If, more generally, $u_k$ is in fact an m-tuple of sub-symbols $u_k^r$ ,

$$t_{Ck} \leq \Sigma_r \, t_{Ck}^r \tag{5.3}$$

A similar relationship can be expected for the error entropies. Thus, if

$$H(v \mid u_k) = - \Sigma_\ell \, p(u_\ell \mid u_k) \log p(v_\ell \mid u_k)$$

is the entropy of Charlie's error probabilities under alternate processing mode, and

$$H(v^1, v^2 \mid u_k) = - \Sigma_{ij} \, p(v_i^1, v_j^2 \mid u_k) \log p(v_i^1, v_j^2 \mid u_k)$$

under the parallel one, the relationship

$$H(v | u_k) \leq H(v^1, v^2 | u_k) \leq H(v^1 | u_k) + H(v^2 | u_k)$$

can be expected to hold because it indicates that operator errors are less likely under first mode than under the second. The second inequality above, utilizes a well-known property of the entropy function ([29], p. 21). More generally, when v is an m-tuple,

$$H(v | u_k) \leq H(v^1, v^2, \ldots, v^m | u_k) \leq \Sigma_r H(v^r | u_k). \tag{5.4}$$

The implications of these relations to organizational design are discussed in Section 6.4.

## 6. DECENTRALIZATION UNDER $\psi$-DEPENDENCE

### 6.1 General Comments

The purpose of this section is to investigate the effect on organizational design of the three motivational factors which have been called $\psi$-factors in the preceding one. The evidence developed below indicates that the first one, namely the members' desire for autonomy, has the most pronounced effect. It reinforces the advantages of decentralization, as well as the trend towards the kind of group operation which was introduced in Section 4.5. In fact, operational procedures emerge as optimal for such groups which seem to resemble at least qualitatively those one would consider good practice in the field. They suggest more particularly the kind of processes that are sometimes referred to as "participative management".

The influence on design of the second $\psi$-factor, namely that of organizational status is perhaps more readily quantifiable but less interesting conceptually. It develops to have a bearing, and perhaps even a strong bearing, on the span of control associated with a supervisory position.

The third factor to be studied is that of job satisfaction. The conclusion reached below is that it is likely to be most pronounced on the lowest line echelon, and that it will favor alternate over parallel processing to the extent to which the time delays often associated with the former are tolerable.

6.2 Effect of Autonomy

The design of an organization semms to be more profoundly affected by the allowances for autonomy among its members, than by those for the other two $\phi$-factors discussed in Section 5 of this report. In other words, if the granting of autonomy induces an improvement in the performance of an organization member, as is generally claimed, certain features become advantageous which otherwise might not be that, or else their advantages are reinforced. Decentralization in particular, is among these features, and so is the group structure described in Section 4.5.

Decentralization was shown in Section 4.2 to be often beneficial to the performance of an organization whose members are load-independent and error-free. However, it was also shown that this need not be so otherwise, as the results presented in Sections 4.3 and 4.4 prove. The first point to be made here is that considerations of autonomy will often reverse that. In particular, the positive effect of autonomy may cancel the negative effect of load-dependence which was shown in Corollary 4.2 to be a potential deterrent to even a slight decentralization. To demonstrate this, suppose that a centralized organization is to be "slightly" decentralized by introducing a by-pass SOP as in (4.1), but by allowing the additional input to come from the private source of the affected line member, rather than from the staff. Thus, under the centralized arrangement,

(6.1a) $$p(u''_o \mid u_j, x_i) = 1, \qquad p(u''_1 \mid u_j, x_i) = 0$$

where x is the input from the "official" source of the organization, u the Executive directive to the line member, $u''_o$ the symbol meaning "no inputs from the private source", and $u''_1$ one other symbol from it. The "slight" decentral-

ization *allows* $u''_1$ *to be sent with probability* $\epsilon$ *when* $x_i$ *and* $u_j$ come through intra-organizational channels. Thus,

$$(6.1b) \qquad p(u''_o \mid u_j, x_i) = 1 - \epsilon \qquad p(u''_1 \mid u_j, x_i) = \epsilon$$

One can then readily prove a result that is a counterpart to Corollary 4.3, namely this.

Proposition 6.1. Suppose that a change in the by-pass SOP from (6.1a) to (6.1b) is carried out. Let the processing lines of the affect line member be $t_{r0}$ under the centralized, and $t'_{r0}$ and $t'_{r1}$ under the decentralized arrangement. Suppose the line member responds to the autonomy implied by the decentralization in such a way that the inequalities (4.5) are reversed, as $\epsilon \to 0$. Then, if (4.2) holds, performance is improved and overload avoided by decentralization.

The proof of this proposition is analogous to that of Corollary 4.3. It can therefore be omitted here. It may be well to point out here, however, that decentralization in this case may eliminate some overload that prevailed in the centralized arrangement.

The preceding result was obtained under the assumption that no cross links exist over which communications could be exchanged among the line. However, as was explained in Section 4.5, organizational performance can often be improved by permitting such communications, notably when the line member has access to private sources and when they do their processing in parallel. One can ask whether or not this conclusion is changed when the autonomy factor is allowed for. It will now be shown that, on the contrary, it is reinforced but that certain changes may be introduced into the mode of operation.

For simplicity, consider once more the two-member line group, shown in Figure 4.3. It illustrates the pattern of communications links that may be necessary for optimal operation of a parallel processing line, if autonomy is disregarded. Proposition 4.6 then points out that cross links are superfluous if the Executive is error-free, and if he remains error-free even when the line members relay to him all of their private inputs. In effect, he himself then supplies those links. In that case, however, Charlie and Delta surrender their autonomy, with the result that their processing times and possibly also their error rates, may increase. Moreover, there is the risk of Executive overload. Both difficulties can be met by a change in the operating mode on which Proposition 4.6 is based.

The way to reduce the Executive's load is to cut back on the transmissions to him from the line. This has the additional advantage of increasing the autonomy among the line. The reduction of the transmissions to the Executive, however, is an equivocation process. This imposes on the line group a kind of task which has so far been associated only with the staff. Part of it is the decision of what to transmit to the Executive. The requirement for the making of this decision induced a special pattern of communications links, a special SOP, and a special rank order among the members of a staff group (described in Section 6.5 of [10]). The same features are now induced also among the members of a line group. The pattern of communications links which is illustrated for such a group in Figure 4.3 is sufficient to accommodate this new task. The rank order, however, poses a disparity in the autonomies among the group members. Thus, if Charlie is the one in the two-member line group who makes the decision of what joint symbol to send to the Executive, Delta is left with no choice of her own and hence with no autonomy in this regard. This will depress her performance, to the detriment of that of the group.

The disparity can be aggravated if the Executive on his part uses an SOP for the distribution of the output load among the line. Instead of assigning to each line member the portion of the output task which he is to perform, he can assign the whole task to one member, Charlie for instance, with understanding that the sub-tasks will be parcelled out among the rest of the group in the way this was envisaged for the staff (in Section 6.5 of [10]). This would decrease his own load, which might be advantageous. It would also increase Charlie's degree of autonomy but Delta's would remain zero.

Considerations of performance, as well as equity, suggest that such disparities in autonomy be avoided. This can be done by an SOP, assigned presumably to the Executive, and which requires him to choose in a suitably randomized fashion among the various rank orders under which the line group can operare. He can randomize the order in which the messages to him are selected by the line, or that in which the line members select the outputs they produce for their destinations. For optimum performance, he will no doubt consider the randomization of both.

One can write down a complete catalog of rank orders for a line group, and for the SOP that goes with each, but it develops to be too messy to be very instructive. The SOP's are all derived by factoring in various ways the joint conditional probability of the outputs, internal as well as external, which the group and the Executive generate. The conditioning is on the inputs they receive. For the two-member group of Figure 4.3, for instance, this probability is (omitting the symbol subscripts, for convenience)

$$p(u, v, y, y', z, z', w, w' \mid x, u'', v'').$$

The factorization that represents, for instance, the operating procedure under which Charlie selects the transmission (z', w') to the Executive but the Executive takes care of the distribution of directives (u,v) among the line, would be

$$(6.2) \quad p(y|u,u'')p(y'|v,v')p(u,v|x,z',w')p(z'|u'',w)p(z|u'')p(w'|v'',z,z')p(w|v'') .$$

The first two factors represent the SOP with which Charlie and Delta choose the outputs for their destinations, and the last four the SOP which they follow in distributing their private inputs to each other and to the Executive. The third factor represents the Executive SOP. All probabilities are those of outputs, conditioned in each case on the inputs on which the output can be based.

Similar expressions can be written down for the seven other operation modes. Under the randomized procedure mentioned above, each is then chosen with a probability $\pi_\nu$, $\nu = 1, 2, \ldots, 8$. One can define the optimal SOP for the line group and the Executive as one which minimizes the largest load among the line and avoids overload on the Executive, on the theory that this will also avoid overload among on the line, if it can be avoided there in the first place. Such SOP's were called "minimax" in Section 5.4 of [10]. In the present case, one can say the following.

Proposition 6.2. Minimax SOP's are among those which equalize the mean processing times of the members of the line group. Such SOP's exist, regardless of whether the line members are autonomy-minded.

The proof of this proposition is very similar to that of Proposition 5.2 of [10]. It will not be elaborated on here.

This proposition may be interest because it deals with an SOP that is broader than those considered in this study so far because it is a mixture of several

other SOP's. It describes a procedure for the conduct of business on the line which is strongly reminiscent of what actually takes place in practice and which is called "participative management" and sometimes also "group think" [1].

The proof of Proposition 4.6, as well as that of its prototype in [10], suggests an algorithm by which the *minimax* SOP can be found, if the probabilities characterizing the component SOP's of the group are known. This includes not only those shown as factors in the expression (6.1) but also the joint probability $p(u'', v'', x)$ of the inputs to the group. These, however, are affected by the SOP's used by preprocessors such as the organizational staff, the Executive, and the Censors. The fact that the line members are autonomy-minded makes their processing times and the error entropies of an organization member functions of these. This is nub of Propositions 5.1 and 5.2. The net effect of this is very much like load-dependence, and more particularly the effect of input load. In fact, under the assumptions of Proposition 5.2 the processing times are concave function of the probabilities, as shown there. They, therefore, have precisely the same properties as those of an organization member who is input load-dependent. It follows that the results derived for such organization members apply also to autonomy-conscious ones. One can, in particular, say the following.

<u>Proposition 6.3.</u> Under the assumptions of Proposition 5.2, the best equivocation SOP for an autonomy-conscious organization member is pure, provided it avoids overload on him.

<u>Proof.</u> It merely needs to be pointed out here, as has been in several other proofs (of [10]), that the optimum SOP for any staff member is pure if it avoids overload in the line member he serves. This conclusion was, however, reached disregarding any effects of autonomy. It is easy to see that it remains valid

also if this effect is taken into account. Suppose the opposite. In that case a departure from a pure SOP would induce an improvement. However, since the mean processing time is concave in the equivocation probabilities of the SOP, the departure would increase it. Hence, overload might be introduced and, with it, a performance deterioration.

This result seems quite consistent with observations in practice. An organization member who is given a certain degree of autonomy in his operation rarely, if ever, needs to contend with random variations in that degree. One can surmise that, if certain random variations could produce tangible performance improvements, there would be at least an effort made in practice to realize them. The fact that this is not done may indicate that the assumptions of Proposition 5.2 are valid. For, if they were not, mixed SOP's would frequently be better than pure ones, and one should expect to encounter them in practice.

## 6.3 Effects of Organizational Status

The discussion of this effect on an individual organization member in Section 5.3, led to the conclusion that it is a vector of at least three factors, of which only two are likely to influence the design of an organization. These were the proportion $\beta_1$ of the time which a supervisor devotes to the member and the proportion $\beta_2$ allocated to him by each of his peers. It was further suggested that $\beta_1$ was the more important of the two factors. The discussion in this section will indicate that the main design effect is the one of span of control of the supervisor.

The time allocations $\beta_1$ and $\beta_2$ are presumably to be used for the exchange of formal and ivformal communications. In the particular case of the

organizational line unit shown in Figure 4.3, these communications would be transmitted over the links shown there and no additional ones would be needed. The group structure that is illustrated there is accordingly sufficient, but any other with fewer links is not. The Designer, therefore, who wishes to arrange for the motivation of the members of an organization by their status factors will thus provide for the group structure, on the line as well as on the staff.

The communications links within the group would then be used not only for the purposes outlined for them in Section 4.5, and in the preceding one, namely by the supervisor for issuing work directives; by all group members for the pooling of the inputs from their private sources; and by the non-supervisory ones possibly also for the coordination of their joint outputs. A supervisor who wishes to utilize the status factor would use those links also for other communications, such as

(a) information regarding the organization, of potential interest to the group;

(b) discussion of the SOP which is to be jointly followed, subject to the constraints laid down under the autonomy rules discussed in Section 5.2;

(c) reviews of individual and joint performance;

(d) advice and help, intended to improve performance;

(e) suggestions, questions, and complaints by group members, and, if indicated by their subordinates;

(f) small talk.

The communications among the peers of a group would be concerned with essentially the same topics.

The main effect on organizational design of the $\beta_1$-factor, i.e. of the supervisory time allocation to each subordinate, would be on the span of control. The larger $\beta_1$, the smaller the span. In fact, one can expect the total allocation to be roughly proportional to the number m of subordinates, which would mean that the span would be

$$m = \beta_1^{-1} \quad ,$$

and that only if the remaining activities of the supervisor consumed a negligible option of his time. More often, it would mean that overload would set in if those remaining activities look up more than $(1 - m\beta_1)$ time units.

In fact, since every supervisor, except the Executive himself, is in turn a member of a group and hence must allocate $\beta_2$ time units, the overload limit will be $(1 - m\beta_1 - (m-1)\beta_2)$. The numerical value of this quantity may depend quite strongly on the nature of the work done by a group, as was intimated at the end of Section 5.3.

## 6.4 The Effect of Job Satisfaction

The third of the $\psi$-factors discussed in Sect. 5 was the job satisfaction factor. It was interpreted there as indicating that the members of the lowest line echelon were often better satisfied with their jobs if they could do them by alternate rather than parallel processing. This observation was expressed mathematically by equations (5.3) and (5.4). The main object here will be to show that these relations in themselves are sufficient to guarantee better performance of the line members individually, but not necessarily jointly. This is the substance of

Proposition 6.3. Suppose that the optimal SOP for an alternately processing line fully loads all members of an m-member line echelon. Assume further that

$$\text{(6.3)} \qquad \min_{\nu} \sum_R t_{Ri}^{\rho_r} > t_{JR}$$

when ranges over all allowed permutations ( $\rho_1$, $\rho_2, \ldots, \rho_m$) of the m-tuple (1,2,...,m) and J and R range over all members of the echelon. In that case, the echelon is overloaded by every parallel processing SOP. On the other hand, if merely

$$\text{(6.4)} \qquad \sum_{\rho} t_{Ri} > t_{Ri} \qquad (R=1,2,\ldots,m),$$

as in (5.3), parallel processing may be superior.

Proof. It will be sufficient to consider a two-member line echelon, consisting of Charlie and Delta. Eq. (6.3) then takes the form

$$t_{Ci}^{\rho_1} + t_{Di}^{\rho_2} > t_{Ci} \;, \quad t_{Ci}^{\rho_1} + t_{Di}^{\rho_2} > t_{Di}$$

for every permutation ( $\rho_1$, $\rho_2$) of the index pair (1,2). Suppose that ( $\rho_1$, $\rho_2$) is chosen with probability $p_{\nu i}$, $\nu = 1,2,\ldots 4$. Then, e.g., in the first inequality above,

$$\Sigma_\nu p_{\nu i} (t_{Ci}^{\rho_1} + t_{Di}^{\rho_2}) > t_{Ci} \quad \Sigma_\nu p_{\nu i} = t_{Ci}$$

because the probabilities $p_{\nu i}$ add up to one (see Sect. 5.4 of [10]). The left-hand side of the equation can also be written

$$\Sigma_\nu p_{\nu i} (t_{Ci}^{\ 1} + t_{Di}^{\ 2}) = \Sigma_\rho p_{Ci}^{\ \rho} t_{Ci}^{\rho} + \Sigma_\rho p_{Di}^{\ \rho} t_{Di}^{\rho},$$

using the notation of [10] (ibid.) in which e.g., $p_{Ci}^{\ 1}$ us the probability of Charlie's having to produce the output component $y_i^{\ 1}$ of $y_i$. Thus,

$$\Sigma_\rho p_{Ci}^{\ \rho} t_{Ci}^{\rho} + \Sigma_\rho p_{Di}^{\ \rho} t_{Di}^{\rho} > t_{Ci}, \tag{6.5}$$

and an analogous inequality with $t_{Di}$ on the right. Multiply both with the probability $p_i$ of the requirement for $y_i$, and add them. This gives, on the left

$$\Sigma_i p_i \Sigma_\rho p_{Ci}^{\ \rho} t_{Ci}^{\rho} + \Sigma_i p_i \Sigma_\rho p_{Di}^{\ \rho} t_{Di}^{\rho} = \tau_C' + \tau_D' \tag{6.6}$$

where $\tau_C'$ and $\tau_D'$ are Charlie's and Delta's mean parallel processing times, respectively. Now multiply this equation first with the probability $p_{Ci}$ of Charlie's being assigned the job of producing $y_i$, then with the corresponding probability $p_{Di}$, and add the two relations obtained in this way. Since

$$p_{Ci} + p_{Di} = 1$$

(6.6) is reproduced. The same operations on the right-hand side of (6.5) give

$$\Sigma_i t_{Ci} p_{Ci} p_i + \Sigma_i t_{Di} p_{Di} p_i = r_C + r_D$$

where $r_C$ and $r_D$ are Charlie's and Delta's mean alternate processing times, respectively. But $r_C = r_D = 1$ because both are assumed fully loaded in this processing mode. Thus,

$$r_C' + r_D' > r_C + r_D = 2$$

which shows that either Charlie or Delta or both are overloaded in the parallel processing mode, as asserted in this proposition. Suppose next, that (6.4) holds in place of (6.3). The parallel processing SOP which minimizes the combined parallel processing times

$$r_C' + r_D' = \Sigma_i p_i \Sigma_\nu p_{\nu i} (t_{Ci}^{\rho_1} + t_{Di}^{\rho_2})$$

of Charlie and Delta is then defined by a set of probabilities $p_{\nu i}$ which are zero for every $\nu$ and i, except those for which $(t_{Ci}^{\rho_1} + t_{Di}^{\rho_2})$ is a minimum. Suppose that

$$t_{Ci}^{1} + t_{Di}^{2} = 0$$

for every i. This is compatible, in principle, with (6.4), and is clearly a minimum. It merely implies that, for all i,

$$t_{Ci}^{2} > t_{Ci}, \qquad t_{Di}^{1} > t_{Di}.$$

In this case, however, the parallel processing SOP which assigns $y_i^1$ to Charlie and $y_i^2$ to Delta with probability one achieves vanishing mean processing times for both. It is therefore patently better than **even** the best alternately processing SOP. The proposition is accordingly proven.

The requirement (6.3), which assures better performance from a line, made up of members who are sensitive to the job satisfaction factor, is quite strong. It says in effect that all members are so displeased under parallel processing that there is no way of apportioning any job among them which would not be done faster by any one of them, processing it alternately. There does not seem to be a plausible and simple way of weakening the requirement, however.

## 7. REFERENCES


1. Anon., Time Magazine, Chicago, January 19, 1976.

2. K.J. Arrow, The Limits of Organization, Norton & Co., New York 1974.

3. L.E. Bjork, An Experiment in Work Satisfaction, Scientific American magazine, 232 (March 1975), p. 17.

4. R.P. Blanc and I.W. Cotton (ed.'s), Computer Networking, IEEE Press, New York, 1976.

5. P.M. Blau and W.R. Scott, Formal Organizations, Chandler Publishing Co., San Francisco, 1962.

6. R. Carzo and J.N. Yanouzas, Formal Organization: A Systems Approach, Irvin-Dorsey Press, Homewood, 1967.

7. N.L. Chevanny and G.H. Dickson, An Experimental Evaluation of Information Overload, Management Science, Vol. 20 (1974) p. 1335.

8. L.E. Davis and J.C. Taylor (ed.'s), Design of Jobs, Penguin Books, New York, 1972.

9. K.W. Deutsch, The Nerves of Government, Free Press, New York, 1966.

10. R.F. Drenick, A Mathematical Theory of Organization, Part I: Centralized Organization, Final Report, Contract N00014-67-A-0438-0012, 1974. (AD-A003601)

11. R.F. Drenick, The Control of Systems in the Presence of Uncertainty, SIAM. Journal for Control and Optimization, Vol. 12, (May 1977).

12. J.W. Fesler, Centralization and Decentralization, in: International Encyclopedia of the Social Sciences, Vol. 2, MacMillan, New York, 1968.

13. P. Fishburn, Utility Theory in Decision Making, Wiley, New York, 1970.

14. J.L. Franklin, A Path-Analytic Approach to Describing Social-Psychological Variables in Multi-Level Organization, Ph. D. Thesis, University of Michigan, 1973.

15. W.R. Garner, Uncertainty and Structure as Psychological Concepts, Wiley, New York, 1962.

16. Y.-C. Ho and K.-C. Chu, Team Decision Theory and Information Structures in Optimal Control Problems, IEEE Trans. Autom. Contr. Vol AC-17(1972) p. 15.

17. R.J. House, et. al., Expectancy Theory as a Predictor of Work Behavior and Attitude: a Re-Evaluation of Empirical Evidence, Decision Sciences, Vol. 5, (1974) p. 481.

18. M. Kochen and K.W. Deutsch, A Note on Hierarchy and Coordination: An Aspect of Decentralization, Management Science, Vol. 21 (1974), p. 106 and earlier papers referenced there.

19. R. Likert, New Patterns in Management, McGraw-Hill, New York 1961.

20. R.D. Luce, et. al., Handbook of Mathematical Psychology, Vol. 1, Wiley, New York, 1963.

21. J. Marschak, and R. Radner, Economic Theory of Teams, Yale University Press, New Haven, 1969.

22. J. Marschak, Optimal Incentives in an Organization, Working Paper No. 256, Western Management Science Inst., 1976.

23. J. Marschak, Oral Comment.

24. T. Marschak, Centralization and Decentralization in Economic Organizations, Econometrica, Vol. 27 (1959) p. 399.

25. J.E. McGrath and M. Altman, Small Group Research: A Synthesis and Critique of the Field, Holt, Reinhart and Winstron, New York, 1966.

26. L.W. Porter, et. al., Behavior in Organizations, McGraw-Hill, New York 1975.

27. A.W. Roberts and D.E. Varberg, Convex Functions, Academic Press, New York, 1973.

28. L.J. Savage, The Foundations of Statistics, Wiley, New York 1954.

29. C.E. Shannon, The Mathematical Theory of Communication, University of Illinois, Urbana, 1949.

30. H.A. Simon, Administrative Behavior, Free Press, New York, 1959.

31. W.E. Souder, Autonomy, Gratification, and R & D Outputs, Management Science, Vol. 20 (1974) p. 1147.

32. R.M. Stogdill, Individual Behavior and Group Achievement, Oxford University Press, 1959.

33. G.B. Strother, Problems in the Development of a Social Science of Organizatior, in: The Social Sciences of Organization (H.J. Leavitt, ed.) Prentice-Hall, Englewood, 1963.

34. A. Wald, Statistical Decision Functions, Wiley, New York, 1950.

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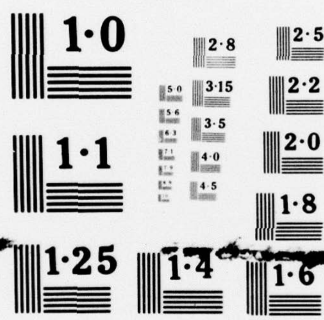


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